HEAT TRANSFER IN FERROFLUID IN CHANNEL WITH POROUS WALLS

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Abstract

The viscous, two-dimensional, incompressible and laminar time dependent heat transfer flow through a ferromagnetic fluid is considered in this paper. Flow takes place in channel between two porous walls under the influence of the magnetic dipole located beyond the channel. It is assumed that there is no electric field effects and the variation in the magnetic field vector that could occur within the ferrofluid is negligible. This magneto-thermo-mechanical problem is governed by dimensionless equations. Results are obtained using standard computational fluid dynamics code COMSOL with modifications to account for the magnetic term when needed.

1 Introduction

In this paper time dependent heat transfer through a ferrofluid in channel flow under the influence of the magnetic dipole is considered and simulated. Results are obtained using standard computational fluid dynamics code COMSOL with modifications to account for the magnetic term when needed.

During the last decades, an extensive research work has been done on the fluids dynamics in the presence of magnetic field. The effect of magnetic field on fluids is worth investigating due to its innumerable applications in wide spectrum of fields. The study of interaction of the magnetic field or the electromagnetic field with fluids have been documented e.g. among nuclear fusion, chemical engineering, medicine, high speed noiseless printing and transformer cooling.

One of the most exciting areas of technology to emerge in recent years is MEMS (micromechanical systems), where engineers design and build systems with physical dimensions in micrometers, e.g. MEMS-based biosensors or microscale heat exchangers. The transport of momentum and energy in miniaturized devices is diffusion limited because of their very low Reynolds numbers. Using ferrofluids in these applications and manipulating the flow of ferrofluids in these applications by external magnetic field can be a viable alternative to enhance convection in these devices.

Ferrofluids are non-conducting fluids and the study of the effect of magnetisation has yielded interesting information. In equilibrium situation the magnetization property is generally determined by the fluid temperature, density and magnetic field intensity and various equations, describing the dependence of static magnetization on these quantities. The simplest relation is the linear equation of state, given by [And1998]. It can be assumed that the magneto-thermo-mechanical coupling is not only described by a function of temperature [And1998], but by an expression involving also the magnetic field strength [Mat1997]. This assumption permit us not to consider the ferrofluid far away from the sheet at Curie temperature in order to have no further magnetization. This feature is essential for physical applications because the Curie temperature is very high (e.g. 1043 Kelvin degrees for iron) and such a temperature would be meaningless for applications concerning most of ferrofluids. So instead of having zero magnetization far away from the sheet, due to the increase of fluid temperature up to the Curie temperature this formulation allows us to consider whatever temperature is desired and the magnetisation will be zero due to the absence of the magnetic field sufficiently far away from the sheet [Tzi2003].

Moreover, ferrofluids are mostly organic solvent carriers having ferromagnetic oxides, acting as solute. Ferrofluids consist of colloidal suspensions of single domain magnetic nanoparticles. They have promising potential for heat transfer applications, since a ferrofluid flow can be controlled by using an external magnetic field [Gan2004]. However, the relationship between an imposed magnetic field, the resulting ferrofluid flow and the temperature distribution is not understood well enough. The literature regarding heat transfer with magnetic fluids is relatively sparse.

An overview of prior research on heat transfer in ferrofluid flows e.g. thermomagnetic free convection, thermomagnetic forced convection and boiling, condensation and multiphase flow are presented in paper [Gan2004]. Many researchers are seeking new technologies to improve the operation of existing oil-cooled electromagnetic equipment. One approach suggested in literature is to replace the oil in such devices with oil-based ferrofluids, which can take advantage of the pre-existing leakage magnetic fields to enhance heat transfer processes. In paper [Tan1999] authors present results of an initial study of the enhancement of heat transfer in ferrofluids in magnetic fields which are steady but variable in space. Finite element simulations of heat transfer to a ferrofluid in the presence of a magnetic field are presented for flow between flat plates and in a box. The natural convection of a magnetic fluid in a partitioned rectangular cavity was considered in paper [Yam2002]. It was found that the convection state may be largely affected by improving heat transfer characteristic at higher Rayleigh number when a strong magnetic field was imposed. The influence of a uniform outer magnetic field on natural convection in square cavity was presented in paper [Kra2002]. It was discovered that the angle between the direction of temperature gradient and the magnetic field influences the convection structure and the intensity of heat flux. Numerical results of combined natural and magnetic convective heat transfer through a ferrofluid in a cube enclosure were presented in paper [Sny2003]. The purpose of this work was to validate the theory of magnetoconvection. The magnetoconvection is induced by the presence of magnetic field gradient. The Curie law states that magnetization is inversely proportional to temperature. That is way the cooler ferrofluid flows in the direction of the magnetic field gradient and displaces hotter ferrofluid. This effect is similar to natural convection were cooler, more dense material flows towards the source of gravitational force. Results were obtained using standard computational fluid dynamics codes with finite element method.

The effect of magnetic field on the viscosity of ferroconvection in an anisotropic porous medium was studied in paper [Ram2004]. It was found that the presence of anisotropic porous medium destabilizes the system, where as the effect of magnetic field dependent viscosity stabilizes the system. In this paper the investigated fluid was assumed to be incompressible having variable viscosity. Experimentally it has been demonstrated in prior research that the magneto viscosity has got exponential variation, with respect to magnetic field. As a first approximation for small field variation, linear variation of magneto viscosity has been used in paper [Ram2004]. The effect of magnetic field dependent (MFD) viscosity (magnetoviscosity) on ferroconvection in a rotating sparsely distributed porous medium has been studied in paper [Vai2002]. The effect of MFD viscosity on thermosolutal convection in ferromagnetic fluid has been considered for a ferromagnetic fluid layer heated and soluted from below in the presence of a uniform vertical magnetic field [Sun2005]. Using the linearized stability theory and the normal mode analysis method, an exact solution was obtained for the case of two free boundaries.

One of the problems associated with drug administration is the inability to target a specific area of the body. Among the proposed techniques for delivering drugs to specific locations within the human body, magnetic drug targeting [Vol2002, Rit2004] surpasses due to its non-invasive character and its high targeting efficiency. Although the method has been proposed almost 30 years ago, the technical problems obstruct possible applications. It was the aim of the paper [Vol2002] to classify the emerging problems and propose satisfactory answers. A general phenomenological theory was developed and a model case was studied, which incorporates all the physical parameters of the problem. A hypothetical magnetic drug targeting system, utilizing high gradient magnetic separation principles, was studied theoretically using FEMLAB simulations in paper [Rit2004]. This new approach uses a ferromagnetic wire placed at a bifurcation point inside a blood vessel and an externally applied magnetic field, to magnetically guide magnetic drug carrier particles through the circulatory system and then to magnetically retain them at a target site.

2 The governing equations

2.1 Magnetostatic and quasi-static fields

Under certain circumstances it can be helpful to formulate the problems of electromagnetic analysis in terms of the electric scalar potential V and the magnetic vector potential $\mathbf{A}$. They are given by the equalities [Kov1990]

$$\mathbf{B} = \nabla \times \mathbf{A}\,,\ \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \tag{2.1}$$

The defining equation for the magnetic vector potential is a direct consequence of the magnetic Gauss' law. The electric potential results from Faraday's law. Using the definitions of the potentials and the constitutive relation $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$, Ampère's law can be rewritten as

$$\sigma \frac{\partial \mathbf{A}}{\partial t} + \nabla \times \left(\mu_0^{-1} \nabla \times \mathbf{A} - \mathbf{M}\right) - \sigma \mathbf{v} \times (\nabla \times \mathbf{A}) + \sigma \nabla V = \mathbf{J}^e. \tag{2.2}$$

The equation of continuity, which is obtained by taking the divergence of the above equation, gives us the equation

$$-\nabla \cdot \left(\sigma \frac{\partial \mathbf{A}}{\partial t} - \sigma \mathbf{v} \times (\nabla \times \mathbf{A}) + \sigma \nabla V - \mathbf{J}^e \right) = 0 \tag{2.3}$$

These two equations give us a system of equations for the two potentials $\mathbf{A}$ and V.

In the static case we have the equations

$$-\nabla \cdot \left(-\sigma \mathbf{v} \times (\nabla \times \mathbf{A}) + \sigma \nabla V - \mathbf{J}^e\right) = 0\,, \tag{2.4}$$

$$\nabla \times \left(\mu_0^{-1} \nabla \times \mathbf{A} - \mathbf{M}\right) - \sigma \mathbf{v} \times (\nabla \times \mathbf{A}) + \sigma \nabla V = \mathbf{J}^e. \tag{2.5}$$

The term $\sigma \mathbf{v} \times (\nabla \times \mathbf{A})$ represents the current generated motion with a constant velocity in a static magnetic field, $\mathbf{J}^B = \sigma \mathbf{v} \times \mathbf{B}^e$. Similarly the term $-\sigma \nabla V$ represents a current generated by a static electric field, $\mathbf{J}^E = \sigma \mathbf{E}^e$.

If $\mathbf{v} = \mathbf{0}$ the equations decouple and can be solved independently. The other formulation is the single equation

$$\nabla \times \left(\mu_0^{-1} \nabla \times \mathbf{A} - \mathbf{M}\right) = \tilde{\mathbf{J}}^e. \tag{2.6}$$

The conductivity cannot be zero anywhere when the electric potential is part of the problem, as the dependent variables would then vanish from the first equation.

Simplifying to a two-dimensional problem with perpendicular currents that are 0 it should be noted that this implies that the magnetic vector potential has a nonzero component only perpendicularly to the plane

$$\mathbf{A} = (0, 0, A_z). \tag{2.7}$$

Ampère's law can be rewritten as

$$\nabla \times \left(\mu_0^{-1} \nabla \times A_z - \mathbf{M}\right) = \mathbf{0}. \tag{2.8}$$

Along a system boundary reasonably far away from the magnet we can apply a magnetic insulation boundary condition $A_z = 0$. Solving equation (2.8) together with the constitutive relation we can get magnetic vector potential. Using $\mathbf{B} = \nabla \times \mathbf{A}$ and the constitutive relation we can get

$$\mathbf{B} = \left(\frac{\partial A_z}{\partial y}, -\frac{\partial A_z}{\partial x}, 0 \right),\ \mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}. \tag{2.9}$$

2.2 The magnetic field intensity

In this paper the considered flow is influenced by magnetic dipole. We assumed that the magnetic dipole is located at distance $|b|$ below the sheet at point (a,b). The magnetic dipole gives rise to a magnetic field, sufficiently strong to saturate the fluid. In the magnetostatic case where there are no currents present, Maxwell-Ampere's law reduces to $\nabla \times \mathbf{H} = \mathbf{0}$. When this holds, it is also possible to define a magnetic scalar potential by the relation $\mathbf{H} = -\nabla V_m$ and its scalar potential for the magnetic dipole is given by [And1998]

$$V_m(\mathbf{x}) = V_m(x_1, x_2) = \frac{\gamma}{2\pi} \frac{x_1 - a}{(x_1 - a)^2 + (x_2 - b)^2} . \tag{2.10}$$

where γ is the magnetic field strength at the source (of the wire) and (a,b) is the position were the source is located.

2.3 Heat transfer and fluid flow

The governing equations of the fluid flow under the action of the applied magnetic field and gravity field are: the mass conservation equation, the fluid momentum equation and the energy equation for temperature in the frame of Boussinesque approximation.

The mass conservation equation for an incompressible fluid is

$$\nabla \cdot \mathbf{v} = 0 . \tag{2.11}$$

The momentum equation for magnetoconvective flow is modified from typical natural convection equation by addition of a magnetic term

$$\rho_0 \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \nabla \cdot \mathbf{S} + \alpha \rho_0 g (T - T_0) \mathbf{k} + (\mathbf{M} \cdot \nabla) \mathbf{B} \tag{2.12}$$

where ρ_0 is the density, $\mathbf{v}$ is the velocity vector, p is the pressure, T is the temperature of the fluid, $\mathbf{S}$ is the extra stress tensor, $\mathbf{k}$ is unit vector of gravity force and α is the thermal expansion coefficient of the fluid.

The energy equation for an incompressible fluid which obeys the modified Fourier's law is

$$\rho_0 c \left(\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = k \nabla^2 T + \eta \Phi - \mu_0 T \frac{\partial \mathbf{M}}{\partial T} \cdot ((\mathbf{v} \cdot \nabla) \mathbf{H}) \tag{2.13}$$

where k is the thermal conductivity, η is the viscosity and $\eta\Phi$ is the viscous dissipation

$$\Phi = \left(2 \left(\left(\frac{\partial v_1}{\partial x} \right)^2 + \left(\frac{\partial v_2}{\partial y} \right)^2 \right) + \left(\frac{\partial v_2}{\partial x} + \frac{\partial v_1}{\partial y} \right)^2 \right) . \tag{2.14}$$

The last term in the energy equation represents the thermal power per unit volume due to the magnetocaloric effects.

2.4 The Kelvin body force for magnetoconvective flow

The last term in the momentum equation represents the Kelvin body force per unit volume

$$\mathbf{f} = (\mathbf{M} \cdot \nabla) \mathbf{B} , \tag{2.15}$$

which is the force that a magnetic fluid experiences in a spatially non-uniform magnetic field. We have established the relationship between the magnetization vector and magnetic field vector

$$\mathbf{M} = \chi_m \mathbf{H} . \tag{2.16}$$

Using the constitutive relation (relation between magnetic flux density and magnetic field vector) we can write the magnetic induction vector in the form

$$\mathbf{B} = \mu_0 (1 + \chi_m) \mathbf{H} . \tag{2.17}$$

The variation of the total magnetic susceptibility χ_m is treated solely as being dependent on temperature [Gan2004]

$$\chi_m = \chi_m(T) = \frac{\chi_0}{1+\alpha(T-T_0)} . \tag{2.18}$$

Finally, the Kelvin body force can be represented by

$$\mathbf{f} = \frac{1}{2} \mu_0 \chi_m (1+\chi_m) \nabla (\mathbf{H}\cdot\mathbf{H}) + \mu_0 \chi_m \mathbf{H}((\mathbf{H}\cdot\nabla)\chi_m) . \tag{2.19}$$

Using equation (2.19) we can write Eq. (2.12) and (2.13) in the form, respectively

$$\begin{aligned} \rho_0\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) &= -\nabla p + \nabla\cdot\mathbf{S} + \alpha\rho_0 g(T-T_0)\mathbf{k} + \\ &+ \frac{1}{2} \mu_0\chi_m(1+\chi_m)\nabla(\mathbf{H}\cdot\mathbf{H}) + \mu_0\chi_m\mathbf{H}((\mathbf{H}\cdot\nabla)\chi_m) \end{aligned} \tag{2.20}$$

and

$$\rho_0 c\left(\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T\right) = k\nabla^2 T + \eta\Phi - \mu_0 T \frac{\partial(\chi_m \mathbf{H})}{\partial T}\cdot((\mathbf{v}\cdot\nabla)\mathbf{H}). \tag{2.21}$$

2.5 The Brinkman equations for porous media flow

Fluid and flow problems in porous media have attracted the attention of industrialists, engineers and scientists from varying disciplines, such as chemical, environmental, and mechanical engineering, geothermal physics and food science. There has been a increasing interest in heat and fluid flows through porous media [Ing2005].

The Brinkman equations describe flow in porous media where momentum transport by shear stresses in the fluid is of importance. The model extends Darcy's law to include a term that accounts for the viscous transport, in the momentum balance, and introduces velocities in the spatial directions as dependent variables. The flow field is determined by the solution of the momentum balance equations in combination with the continuity equation

$$\rho_0 \frac{\partial \mathbf{v}_B}{\partial t} = -\nabla p_B + \nabla\cdot\mathbf{S}_B + \frac{\eta}{k_p}\mathbf{v}_B + \mathbf{F} \tag{2.22}$$

where η is the viscosity, k_p is the permeability of the porous structure (unit: m^2).

The Brinkman equations applications is of great use when modelling combinations of porous media and free flow. The coupling of free media flow with porous media flow is common in the field of chemical engineering. This type of problems arises in filtration and separation and in chemical reaction engineering, for example in the modelling of porous catalysts in monolithic reactors.

Flow in the free channel is described by the Navier-Stokes equations and the mass conservation equation described in previous sections. In the porous domain, flow is described by the Brinkman equations according

$$\rho_0 \frac{\partial \mathbf{v}_B}{\partial t} = -\nabla p_B + \nabla\cdot\mathbf{S}_B + \frac{\eta}{k_p}\mathbf{v}_B \tag{2.23}$$

and

$$\nabla\cdot\mathbf{v}_B = 0 . \tag{2.24}$$

3 The dimensionless equations

For simplicity the preferred work choice is to work in non-dimensional frame of reference. Now some dimensionless variables will be introduced in order to make the system much easier to study [Str2005]. Moreover some of the dimensionless ratios can be replaced with well-known parameters: the Prandtl number Pr, the Rayleigh number Ra, the Eckert number Ec, the Reynolds number Re, the Darcy number Da and the magnetic number Mn, respectively:

$$Pr = \frac{\eta_0}{\rho_0 \kappa},\ Ra = \frac{\alpha \rho_0 g h^3 \delta T}{\eta_0 \kappa},\ Ec = \frac{v_r^2}{c \delta T} = \frac{\kappa^2}{c \delta T h^2},\ Re = \frac{h \rho_0 v_r}{\eta_0} = \frac{\rho_0 \kappa}{\eta_0},\ Da = \frac{h^2}{k_p}, \\ Mn = \frac{\mu_0 H_r^2}{\rho_0 v_r^2} = \frac{\mu_0 H_r^2 h^2}{\rho_0 \kappa^2}. \tag{3.1}$$

Since now primes will not be written (old variables symbols will be used) but it is important to remember that they are still there. The dimensionless form of Navier-Stokes (2.20) and thermal diffusion (2.21) equations are as follows:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + Ra\, Pr \left(T - \frac{T_0}{\delta T} \right) \mathbf{k} + Pr \nabla \cdot \mathbf{S} + Mn\, \mathbf{f} \tag{3.2}$$

and

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T + Pr\, Ec \eta \Phi + Mn\, Ec\, T \frac{\partial (\chi_m \mathbf{H})}{\partial T} \cdot ((\mathbf{v} \cdot \nabla) \mathbf{H}) \tag{3.3}$$

where

$$\mathbf{f} = \frac{1}{2} \chi_m (1 + \chi_m) \nabla H^2 + \chi_m \mathbf{H} ((\mathbf{H} \cdot \nabla) \chi_m) \tag{3.4}$$

and

$$\chi_m = \chi_m (T(\mathbf{x})) = \frac{\chi_0}{1 + (\alpha \delta T) \left(T(\mathbf{x}) - \frac{T_0}{\delta T} \right)}. \tag{3.5}$$

Dimensionless Brinkman equations are as follows

$$\frac{\partial \mathbf{v}_B}{\partial t} = -\nabla p_B + Pr \nabla \cdot \mathbf{S}_B + Pr\, Da\, \mathbf{v}_B. \tag{3.6}$$

In the presents of magnetic field Kelvin body force is added

$$\frac{\partial \mathbf{v}_B}{\partial t} = -\nabla p_B + Pr \nabla \cdot \mathbf{S}_B + Pr\, Da\, \mathbf{v}_B + Mn\, \mathbf{f}. \tag{3.7}$$

4 Numerical solution and conclusions

In this section we present numerical simulation results of heat transfer in ferrofluid. The flow takes place in channel and in channel with porous walls. The two-dimensional time dependent flows are assumed viscous, incompressible and laminar. Above the channel magnetic dipole is located. The fluid is assumed to be electrically nonconducting. It is assumed also that there is no electric field effects. This magneto-thermo-mechanical problem is governed by dimensionless equations (3.2-3.7).

4.1 Heat transfer in ferrofluid in channel

Considered flow takes place in channel between two parallel flat plates. The length of the channel is L and distance between plates is h.
The corresponding boundary conditions for dimensionless variables are assumed:

- For the upper wall $(0 \le x \le L, y = 1)$: the upper wall temperature is kept at constant temperature $T_u / \delta T$. The velocity is 0 (no slip condition).
- For the lower wall $(0 \le x \le L, y = 0)$: the lower wall temperature is kept at constant temperature $T_l / \delta T$. The velocity is 0 (no slip condition).
- For inlet (the left wall) $(x = 0, 0 \le y \le 1)$: the temperature is varying linearly from $T_l / \delta T$ to $T_u / \delta T$ and is given by equation $T_{in} = \frac{T_u - T_l}{\delta T} y + \frac{T_l}{\delta T}$ where $\delta T = |T_u - T_l|$. There is a parabolic laminar flow profile given by equation $u_{in} = -4 \frac{u_0}{u_r} y(y-1)$ for $y \in \langle 0,1 \rangle$ at the inlet end.
- For outlet (the right wall) $(x = L, 0 \le y \le 1)$: the convective flux is assumed for temperature, $\mathbf{n} \cdot (-k \nabla T) = 0$. Pressure outlet is also assumed, $(-p\mathbf{I} + \mathbf{S})\mathbf{n} = -p_0 \mathbf{n}$, where p_0 is the dimensionless atmospheric pressure.

The following initial conditions for dimensionless variables are assumed: the fluid is motionless, the pressure is zero and the temperature is varying linearly from lower to upper wall.

The time-dependent flow is considered for dimensionless time $t \in \langle 0, 0.5 \rangle$. The problem is solved with COMSOL code using direct UMFPACK linear system solver. Relative and absolute tolerance used in calculations are 0.05 and 0.005, respectively.

The following values of temperatures are assumed $T_l = T_0$, $T_u = T_0 + \delta T$ where $T_0 = 300K$ and $\delta T = 30K$.

TABLE 1 THE QUANTITIES FOR FERROFLUID FLOWS

Quantity	Flow A	Flow B	Flow C	Flow D
H_r	**3819.71**	**5092.95**	**9549.29**	**12732.39**
Mn	**1.6961e+7**	**3.0154e+7**	**1.0601e+8**	**1.8846e+8**
Pr	1.4	1.4	1.4	1.4
Ra	2.5701e+7	2.5701e+7	2.5701e+7	2.5701e+7
Ec	2.1810e-12	2.1810e-12	2.1810e-12	2.1810e-12
Re	0.7142	0.7142	0.7142	0.7142

It can be observed that the maximum value of the magnitude of the velocity field of the flow in the channel under the magnetic dipole increases due to the value of the magnetic number.

The flow was relatively uninfluenced by the magnetic field until its strength was large enough for the Kelvin body force to overcome the viscous force. It can be observed that the cooler ferrofluid flows in the direction of the magnetic field gradient and displaced hotter ferrofluid (Fig. 1,2). This effect is similar to natural convection where cooler, more dense material flows towards the source of gravitational force. Ferrofluids have promising potential for the heat transfer applications because a ferrofluid flow can be controlled by using an external magnetic field.

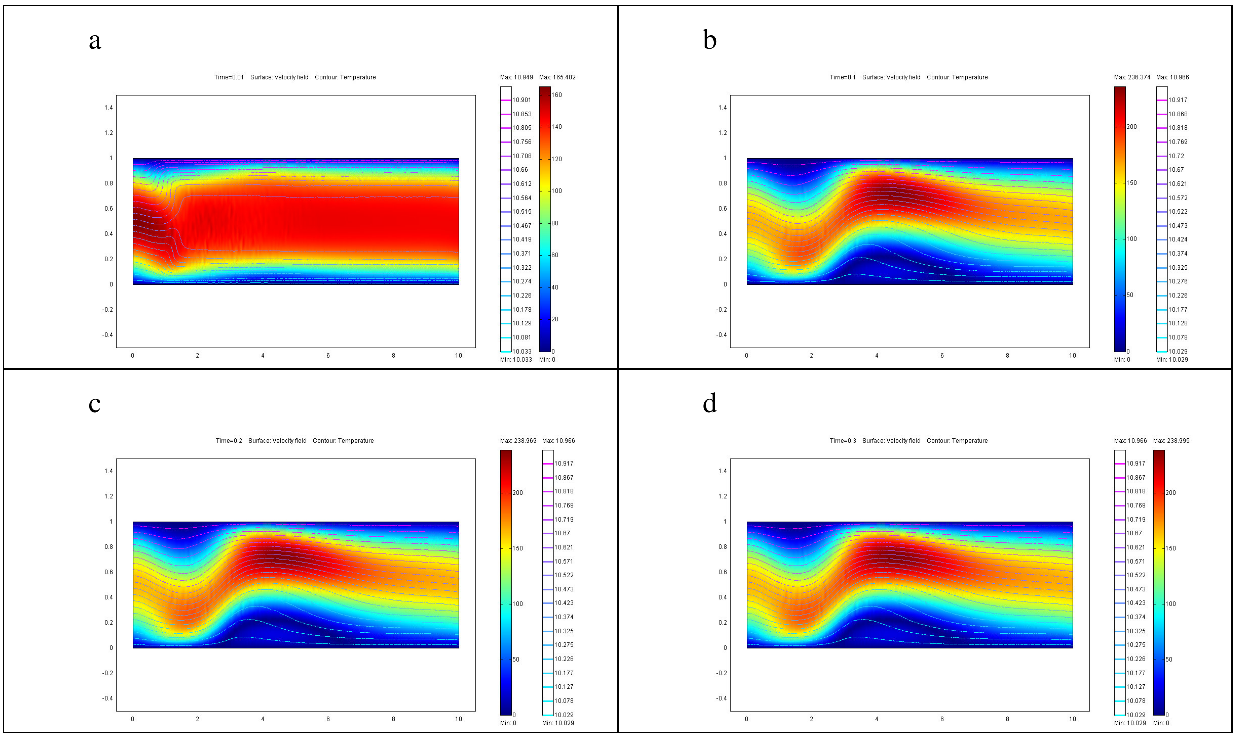


Figure 1. Time evolution of dimensionless velocity field (surface) and temperature contour of flow B for (a) t=0.01 (b) t=0.1 (c) t=0.2 C (d) t=0.3.

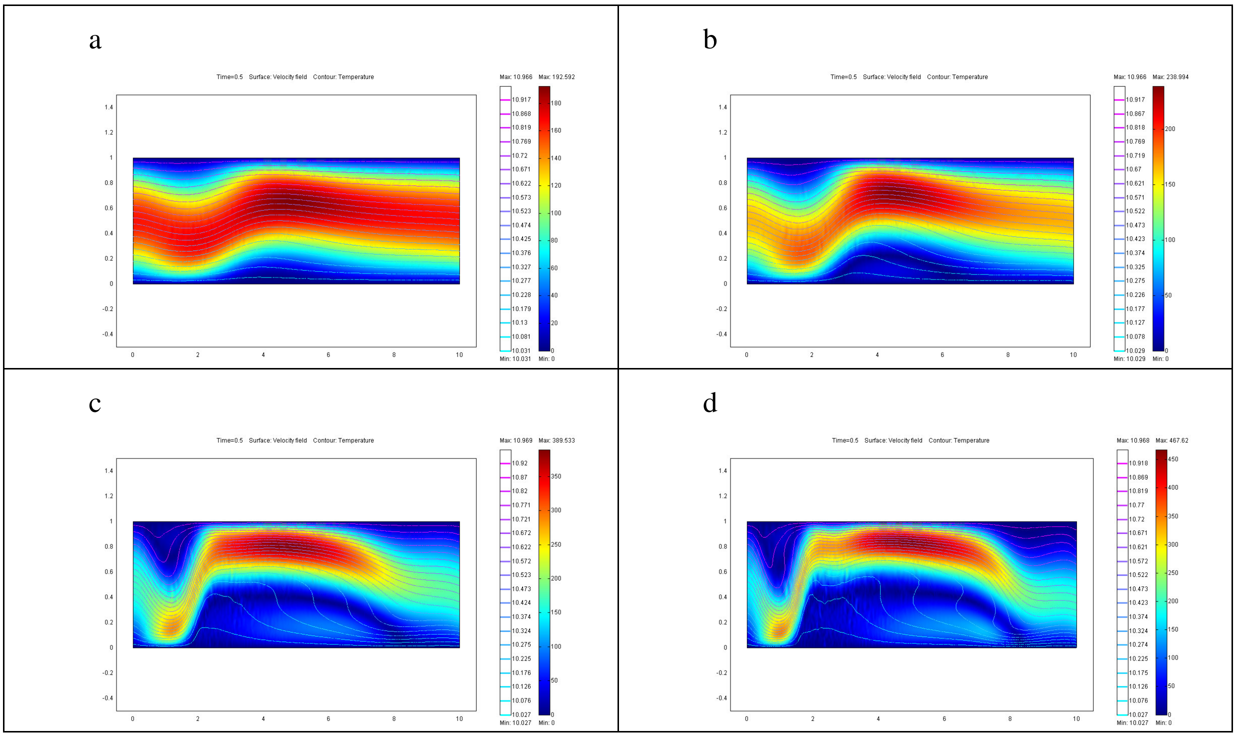


Figure 2. Comparison of dimensionless velocity field (surface) and temperature contour for the different values of magnetic number: (a) flow A (b) flow B (c) flow C (d) flow D.

4.2 Heat transfer in ferrofluid in channel with porous walls

Flow takes place in channel between two parallel porous domains. The length of the channel is L and distance between porous domains is h. The length of porous domains are L and the height are $4h/10$.

The corresponding boundary conditions for dimensionless variables in channel flow are assumed:

- For free-porous structure interface - the upper and the lower wall $(0 \le x \le L, y = 1, y = 0)$, free-porous structure interface: $p = p_B$. The expression for the pressure at the boundary between the channel and the porous domain states that the pressure is continuous across this interface.
- For inlet (the left wall) $(x = 0, 0 \le y \le 1)$: the temperature is varying linearly from $T_l / \delta T$ to $T_u / \delta T$ and is given by equation $T_{in} = \frac{T_u - T_l}{\delta T} y + \frac{T_l}{\delta T}$ where $\delta T = |T_u - T_l|$. There is a parabolic laminar flow profile given by equation $u_{in} = -4 \frac{u_0}{u_r} y(y-1)$ for $y \in \langle 0,1 \rangle$ at the inlet end.
- For outlet (the right wall) $(x = L, 0 \le y \le 1)$: the convective flux is assumed for temperature, $\mathbf{n} \cdot (-k \nabla T) = 0$. Pressure outlet is also assumed, $(-p\mathbf{I} + \mathbf{S})\mathbf{n} = -p_0 \mathbf{n}$, where p_0 is the dimensionless atmospheric pressure.

The corresponding boundary conditions for dimensionless variables in porous domain are assumed:

- For free-porous structure interface: $\mathbf{v}_B = \mathbf{v}$. These conditions imply that the components of the velocity vector are continuous over the interface between the free channel and the porous domain.
- For the upper domain walls: the temperature is kept at constant temperature $T_u / \delta T$. The velocity is 0 (no slip condition).
- For the lower domain walls: the temperature is kept at constant temperature $T_l / \delta T$. The velocity is 0 (no slip condition).

The following initial conditions for dimensionless variables are assumed: the fluid is motionless, the pressure is zero and the temperature is $T_l / \delta T$.

The time-dependent flow is considered for dimensionless time $t \in \langle 0, 0.1 \rangle$. The problem is solved with COMSOL code using direct UMFPACK linear system solver. Relative and absolute tolerance used in calculations are 0.05 and 0.005, respectively.

The following values of temperatures are assumed $T_l = T_0$, $T_u = T_0 + \delta T$ where $T_0 = 300K$ and $\delta T = 30K$.

The heat transfer in ferrofluid flowing in channel with porous walls is considered in four different flows with different magnetic susceptibility, inlet velocity or permeability of the porous structure. The most interesting example of flow we can observe in the last considered flow (flow H). In this case the magnetoconvection is observe (Figure 4-5). We observe vortex created near the centre of magnetic dipole. Each vortex is moving from left to right where the magnetic field intensity is getting smaller. The intensity of magnetic field is plotted on each figure presented in this subsection as contour lines.

TABLE 2 THE QUANTITIES FOR FERROFLUID FLOWS

Quantity	Flow E	Flow F	Flow G	Flow H
H_r	12732.39	12732.39	12732.39	12732.39
χ_0	**0.06**	**0.06**	**0.01**	**0.01**
u_0	**5e-3**	**5e-4**	**5e-2**	**5e-4**
Mn	1.8846e+8	1.8846e+8	1.8846e+8	1.8846e+8
Pr	1.4	1.4	1.4	1.4
Ra	2.5701e+7	2.5701e+7	2.5701e+7	2.5701e+7
Ec	2.1810e-12	2.1810e-12	2.1810e-12	2.1810e-12
Re	0.7142	0.7142	0.7142	0.7142
Da	**4000**	**4000**	**40**	**40**

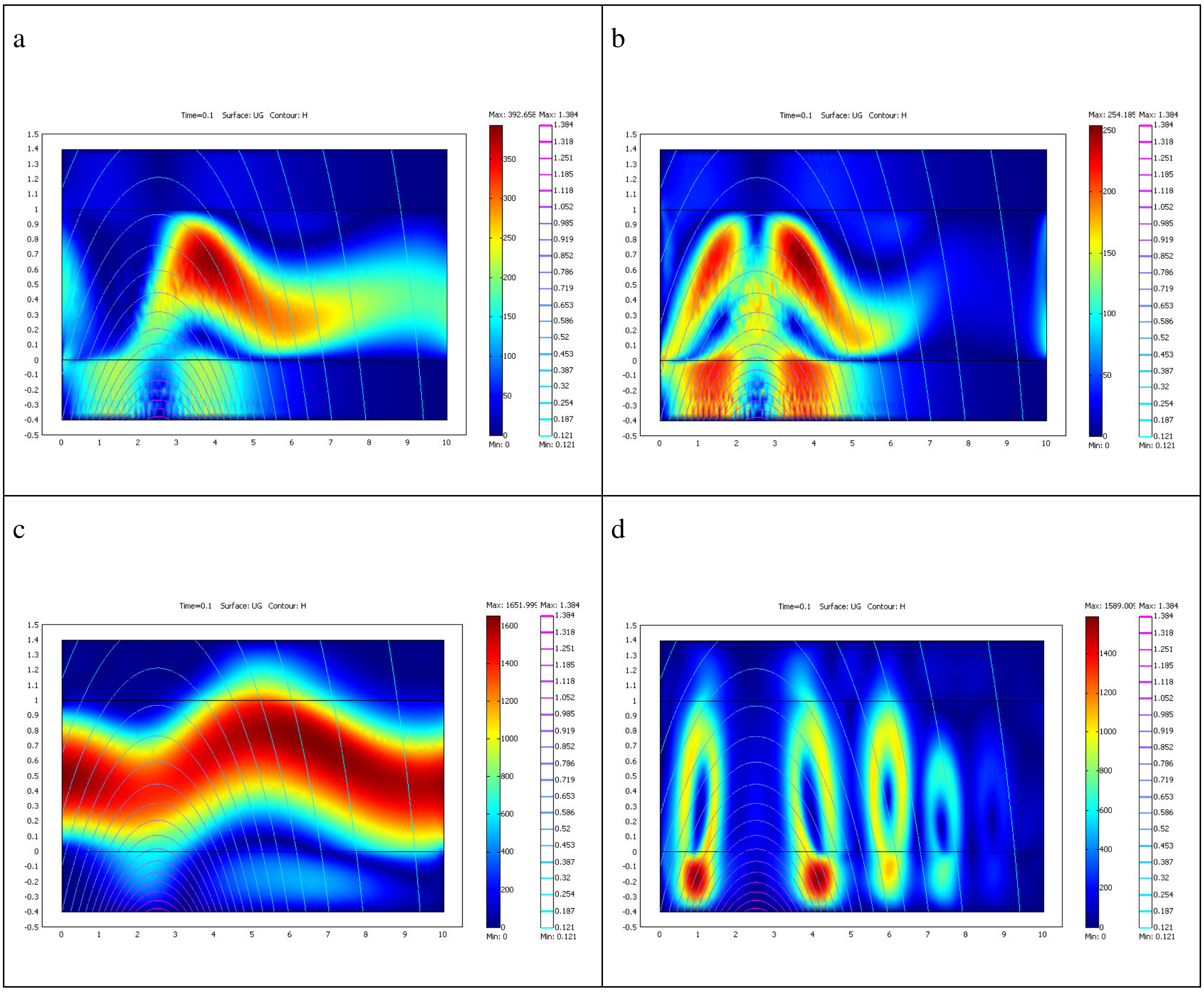
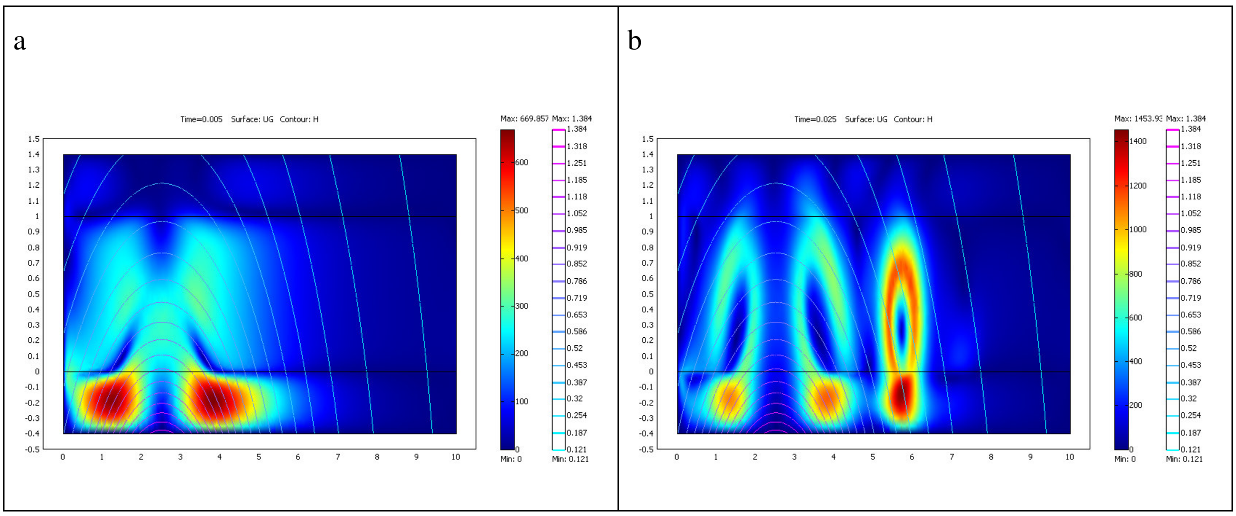


Figure 3. Comparison of dimensionless velocity field for the different flows in channel with porous walls: (a) flow E (b) flow F (c) flow G (d) flow H for time $t = 0.1$.



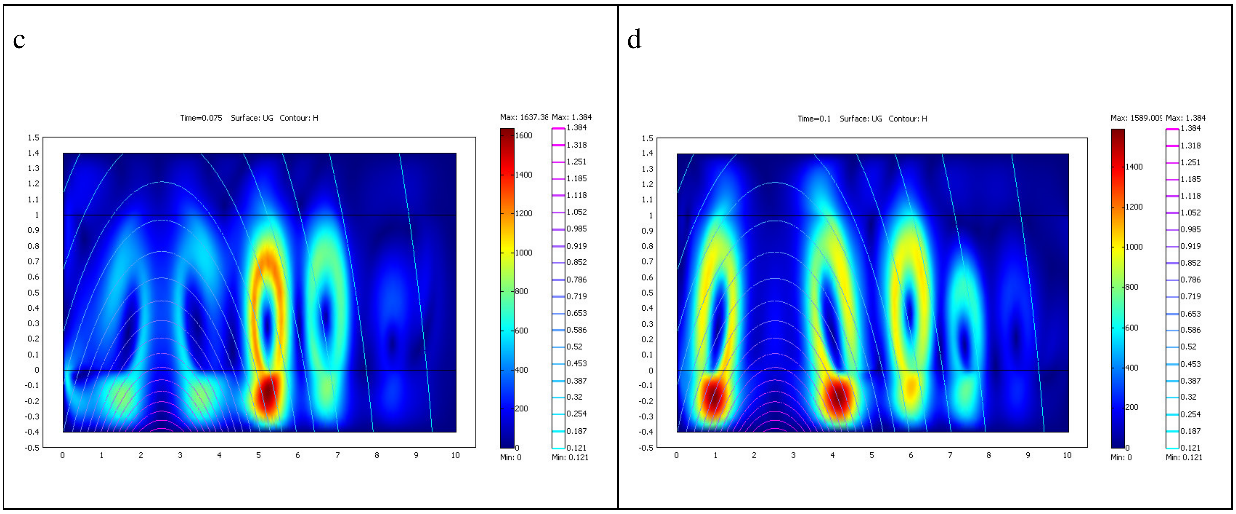


Figure 4. Time evolution of dimensionless velocity field (surface) of flow H for (a) t=0.005 (b) t=0.025 (c) t=0.075 C (d) t=0.1.

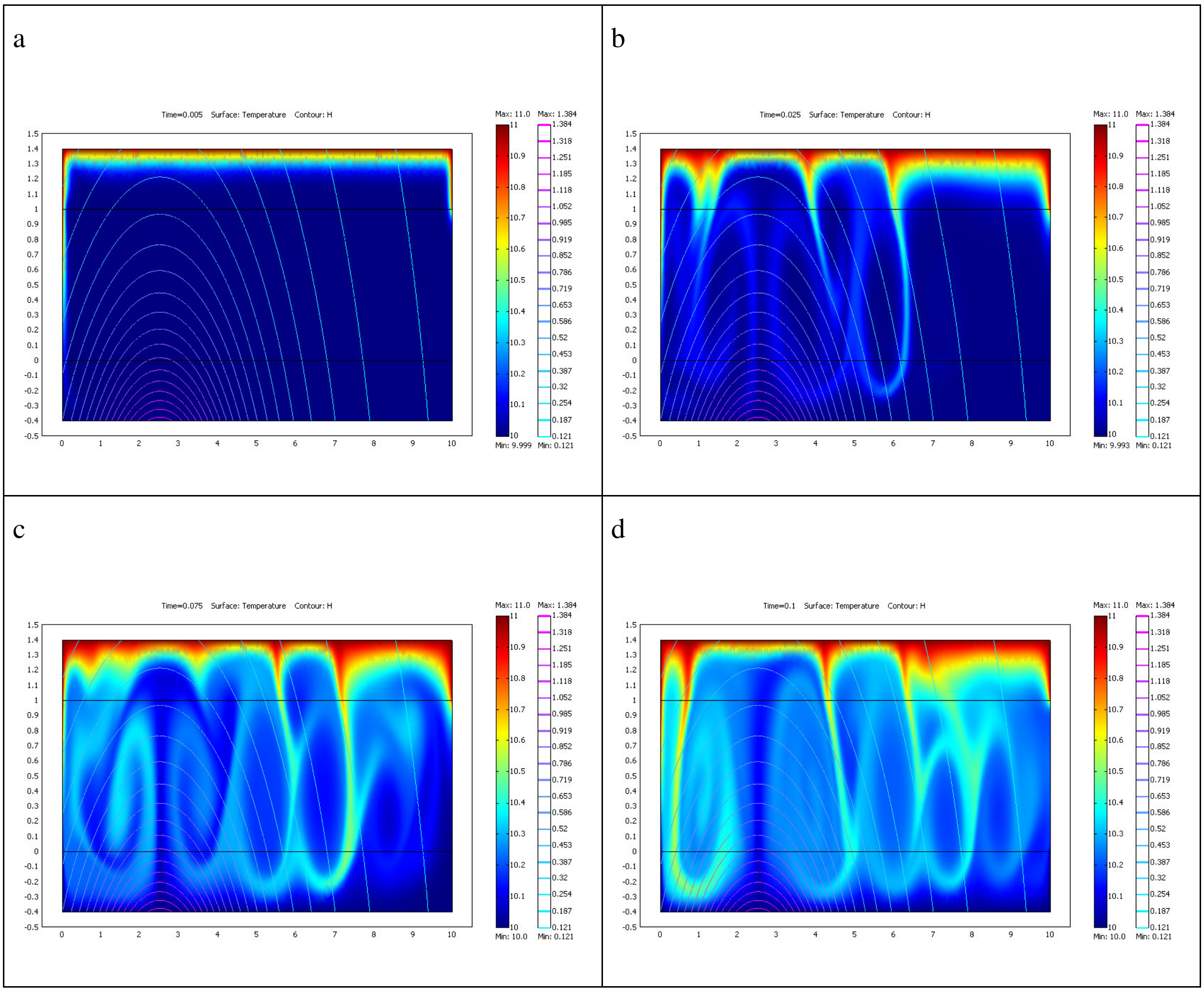


Figure 5. Time evolution of dimensionless temperature (surface) of flow H for (a) t=0.005 (b) t=0.025 (c) t=0.075 C (d) t=0.1.

5 References

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