Risk-Neutral Options Pricing,Implied Distribution of CEZ

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MODERNÍ NÁSTROJE PRO FINANČNÍ ANALÝZU A MODELOVÁNÍ

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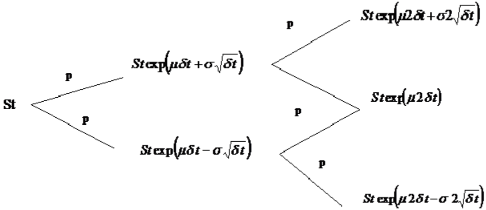
Real World

- Real world $\rightarrow \mathbb{P}$ measure
- Constant grow rate μ
- Constant noise σ
- All jumps equally likely $\rightarrow p = \frac{1}{2}$
- Irrelevant for correct pricing



Risk-Neutral World

- Goal: finding measure $\mathbb{Q}$ under which the process of discounted stock $\left\{B_t^{-1} S_t, t \geq 0\right\}$ is a martingale
- New measure: $q_t = \frac{S_t \exp\left(r\delta t - S_{t+\delta t}^{down}\right)}{S_{t+\delta t}^{up} - S_{t+\delta t}^{down}}$
- Stock value $\rightarrow S_t = S_0 e^{\left(\mu + \sigma\sqrt{t}\left(\frac{2X_n - n}{\sqrt{n}}\right)\right)}$
- $X_n \rightarrow$ number of up-jumps, binomially distributed

Continuous World

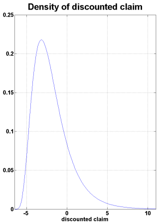
- Transition from discrete to continuous time
- $\delta t \to 0$, $n \to \infty$
- Applying central limit theorem
- S_t log-normally distributed under $\mathbb{Q}$
- $S_t = S_0 e^{\left(\left(r-\frac{1}{2}\sigma^2\right)t+\sigma\sqrt{t}Z\right)}$

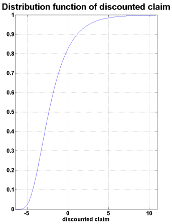
General Pricing Formula

- General pricing formula: $V_0 = E_{\mathbb{Q}}\left(B_T^{-1} X\right)$
- Goal: determining density of discounted claim X
- Transformation of random variable S_T
- Using MATLAB Symbolic Toolbox

Call Option

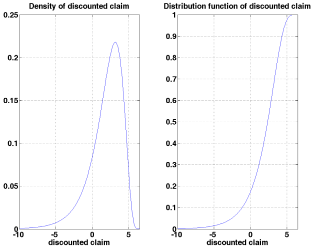
- European call: right, but not obligation, to buy the stock at specified price (strike) at time $t = T$
- Claim: $X = (S_T - k)^+$
- Option value: $V(S_0\,, T) = B_T^{-1} \int_0^\infty x\, f(x) dx$





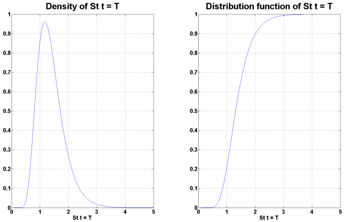
Put Option

- European put: right, but not obligation, to sell the stock at specified price (strike) at time $t = T$
- Claim: $X = (k - S_T)^+$
- Value: $V(S_0, T) = B_T^{-1} \int_0^\infty x\, f(x) dx$



Arbitrary Derivative

- European style
- Claim: $X = min\left\{max\left\{1.3S_0, 0.9\frac{S_T}{S_0}\right\}, 1.8S_0\right\}$
- Numerical integration

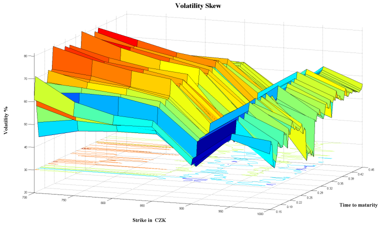


Volatility Smile

- Implied or historical volatility used in pricing formulas
- Implied volatility is unobservable variable
- Nonlognormality can be often observed in the market
- Volatility smiles are used to allow for nonlognormality
- Volatility: function of strike and time to maturity

Volatility Smile on CEZ call

- Call warrants on CEZ stock traded on Boerse Stuttgart denominated in EUR as of 30 April 2009 used
- American exercising style
- Dividend paying



Implied Volatility

- $V_0 = k\omega_1(\delta_k) - S_0\omega_2\left(\delta_{S_0}\right)$

$$\omega_1(\delta_k) = e^{-\delta_k \Delta_t} N_1\left(d_1\left(\frac{Pe^{-\delta\Delta_t}}{P_1^*}\right)\right) + \\ + e^{-\delta_k 2\Delta_t} N_2\left(-d_1\left(\frac{Pe^{-\delta\Delta_t}}{P_1^*}\right), d_1\left(\frac{Pe^{-\delta 2\Delta_t}}{P_2^*}\right); -\sqrt{\frac{t_1}{T}}\right) + \ldots + \\ + e^{-\delta_k \tau \Delta_t} N_n\left(-d_1\left(\frac{Pe^{-\delta\Delta_t}}{P_1^*}\right), \ldots, -d_1\left(\frac{Pe^{-\delta(n-1)\Delta_t}}{P_{n-1}^*}\right), d_1\left(Pe^{-\delta\tau}\right); \Omega_n\right)$$

$$P_1^*\left[e^{-\delta_k\Delta_t}N_1\left(d_1\left(\frac{P_1^* e^{-\delta\Delta_t}}{P^*}\right)\right) + e^{-\delta_k 2\Delta_t}N_2\left(-d_1\left(\frac{P_1^* e^{-\delta\Delta_t}}{P^*}\right), d_1\left(P_1^* e^{-\delta 2\Delta_t}\right); -\sqrt{\frac{t_1}{T}}\right)\right] - \\ - \left[e^{-\delta_d\Delta_t}N_1\left(d_2\left(\frac{P_1^* e^{-\delta\Delta_t}}{P^*}\right)\right) + e^{-\delta_d 2\Delta_t}N_2\left(-d_2\left(\frac{P_1^* e^{-\delta\Delta_t}}{P^*}\right), d_2\left(P_1^* e^{-\delta 2\Delta_t}\right); -\sqrt{\frac{t_1}{T}}\right)\right] \\ = P_1^* - 1,$$

$$d_1(G) = \left(\frac{ln(G) + \frac{\sigma^2\tau}{2}}{\sqrt{\sigma^2\tau}}\right)$$

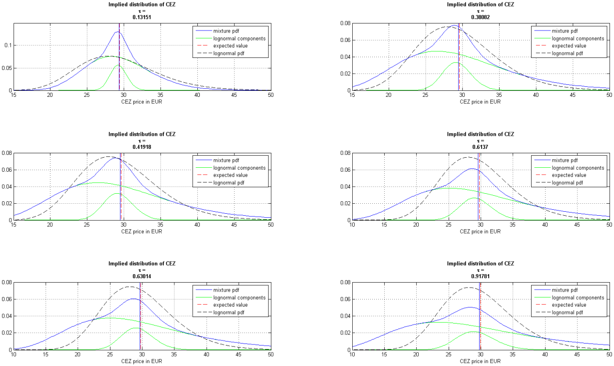
- see [P. Carr, “The Valuation of American Exchange Options with Application to Real Options”] for details

Implied Distribution

- Derived from the market prices of warrants
- Two-lognormal mixture distribution used
- Minimisation problem $\rightarrow$ find parameters of two-lognormal mixture so that the difference between observed prices and implied prices is minimal.
- Trading volume used as weights
- Problem: resulting distribution fulfills the definition of density (integrated to one)
- To avoid arbitrage opportunities: mean of implied distribution must be equal to forward price of CEZ
- KNITRO third-party libraries and MATLAB Optimization Toolbox used

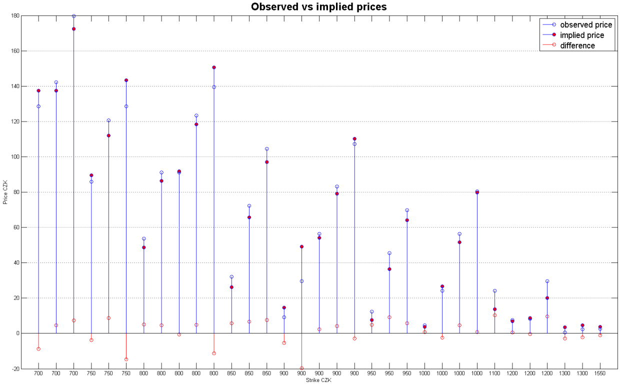
Implied Density of CEZ

- Implied density of CEZ for set of times to maturity τ



Observed vs Implied Option Prices

- Observed vs implied prices by two-lognormal mixture



What for?

- Pricing the options on CEZ not traded on the market
- Searching for mispriced options
- Problem: bid-offer spread

Thank you.

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