Yield Curve Construction and Medium-Term Hedging in Countries with Underdeveloped Financial Markets

Model based pricing and valuation

M. Cincibuch[1] A. Remo[2]

[1]OGResearch

[2]CERGE-EI

June 4, 2010

Objective

Pricing and valuation of

- local currency fixed-rate deals,
- fixed-margin component of floating-rate contracts.

on underdeveloped financial markets.

Construct the term structure of interest rates (discount factors) that take into account risks involved and also compensation for the risk.

Software: Iris

Iris is a Matlab toolbox developed by Jaromír Beneš (www.iris-toolbox.com).
Estimation, interpretation of history, forecasting, forecast integration, decompositions, and much more.
It supports:

- Solving rational expectations models
- Both stochastic and perfect-foresight assumptions
- Identification in both time and frequency domain
- Database and time series handling

Forecasting tools

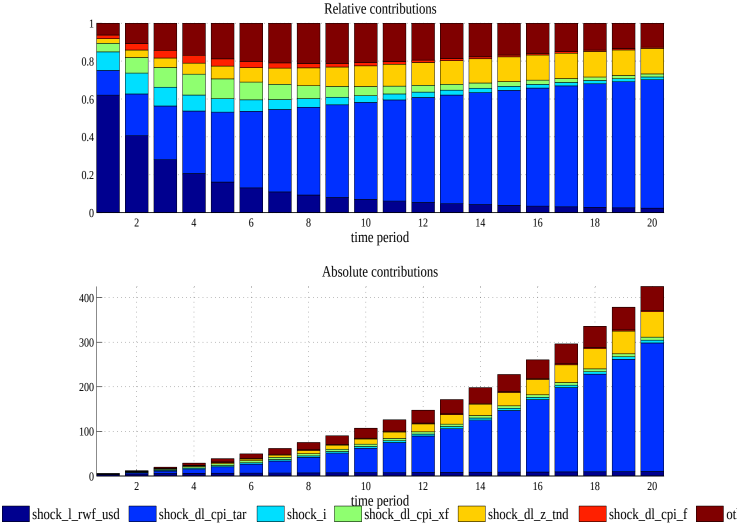
- Structural medium-term macroeconomic model ▸ model equations
  - Flexible NK model of monetary transmission
  - A standard model adapted to EM phenomena
  - Used in practice by many EM central banks
- Near-term forecasting tools
  - Short-term models (ARIMA, VECMs, (B)VARs etc)
  - Expert judgment and sector specific analysis
  - Data driven (black-box) techniques inferior to expert judgment in emerging markets

Forecast based pricing

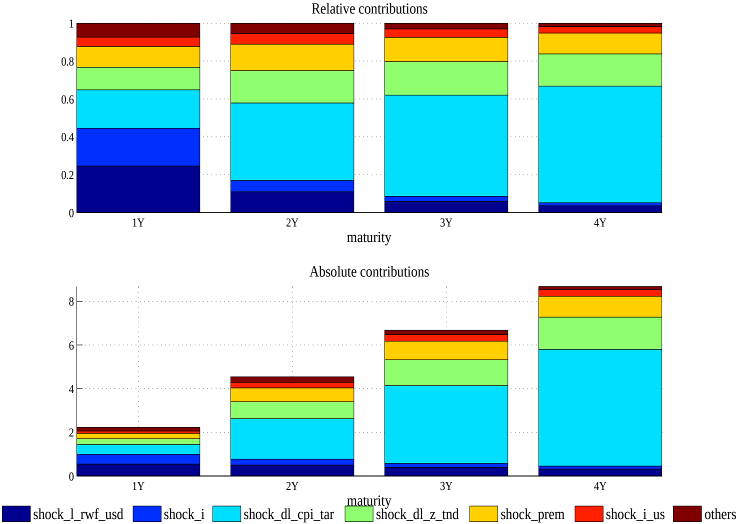
Use medium-term macroconomic projection providing

- forecast of short-term interest rate
- forecast of the exchange rate
- forecast of the other variables
- estimate of the current disequilibrium
- probabilistic distributions for the forecasts

Conditional variance of RWF/USD (100*log)

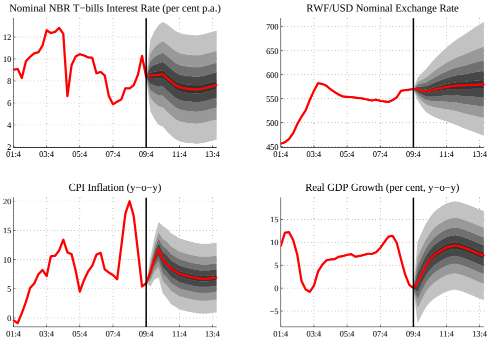


Conditional variance of zero-profit curve - Rwanda



Projection with confidence bands

- Ratios of domestic shock variances calibrated
- Total variance of the model estimated in two steps:
  - Variance of external schocks independently
  - Subsequently the total model variance by ML
- The central projection independent of the band-width.



Different approaches

Two risk based (i.e. whole forecast distribution matters) approaches:

1. Compensates only for local interest rate risk. Based on a local currency float-to-fix swap (IRS).
2. Compensates also for exchange rate risk, foreign term premium. Based on cross-currency float-to-float and fix-to-fix or float-to-fix swaps.

Measure of the price of risk

- Trade-off between risk and return of some zero-investment position measured by the ratio of its expected present value to the standard deviation of this present value.
- It is one possible generalization of the Sharpe ratio (SR) for positions involving cash flow in multiple points of time. For a single period position this coincides with the Sharpe ratio.

Compensating interest rate risk

- The fixed leg rate of local float-to-fix swap set to compensate the interest rate risk.
- The interest rate risk is calibrated on some exogenous reference value of the price of risk.
- Does not include the exchange rate risk directly.

Compensating interest rate risk (cont)

The hypothetical zero-investment position is to roll-over short-term borrowing for T periods in a local currency for $\tilde{r}_t$ and to receive the fixed rate R^T.

The rate R^T is determined by condition $\frac{E(\tilde{PV}_{is})}{std(\tilde{PV}_{is})} = SR_{ref}$:

$$\frac{\left(1+R^T\right)^T - E\left(\left(1+\tilde{C}\right)^T\right)}{\mathrm{std}\left(\left(1+\tilde{C}\right)^T\right)} = SR_{ref},$$

where

$$\left(1+\tilde{C}\right)^T = \Pi_{t=1}^T \left(1+\tilde{r}_t\right)$$

$$\tilde{PV}_{is} = D\left(T\right)\left(N\left(1+R^T\right)^T - N\left(1+\tilde{C}\right)^T\right).$$

Compensating exchange rate risk

- The long-term rate set to compensate also for the exchange rate risk.
- Determined from cross-currency swap's present value distribution and the price of risk.
- A reference value for the price of the risk derived from a *similar* contract for which we have either observable or directly modeled characteristics $\rightarrow$ e.g. basis swap.
- Term premium compensates for additional volatility of the fix-to-fix swap's PV comparing to basis swap.
- Co-movements of the exchange rate and the interest rate decrease the risk of the basis swap.
- Discount factors can be derived from swap rates (bootstrapping).

Compensating exchange rate risk (cont.)

Present values of the basis swap and fix-to-fix swaps are:

$$\tilde{PV}_{basis} = \sum_{t=1}^{T} B^*(t)\left(\frac{N^* S_0 \tilde{r}_{t-1}}{\tilde{S}_t} - N^* \tilde{r}^*_{t-1}\right) + B^*(T)\left(\frac{N^* S_0}{\tilde{S}_T} - N^*\right)$$

$$\tilde{PV}_{fix2fix} = \sum_{t=1}^{T} B^*(t)\left(\frac{N^* S_0 R^T}{\tilde{S}_t} - N^* R^{T*}\right) + B^*(T)\left(\frac{N^* S_0}{\tilde{S}_T} - N^*\right),$$

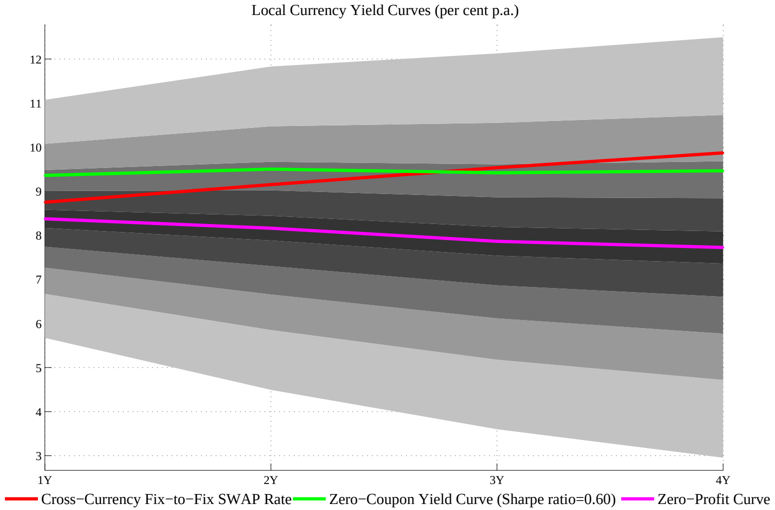
The rate R^T of the tenor T is determined by condition:

$$\frac{E\left(\tilde{PV}_{fix2fix}\right)}{std\left(\tilde{PV}_{fix2fix}\right)} = \frac{E\left(\tilde{PV}_{basis}\right)}{std\left(\tilde{PV}_{basis}\right)}.$$

Presented prototype yield curves are based on the January 2010 Rwanda forecast:

- Projected short-term rate was compounded to get the base *zero-profit* curve - shown as violet line.
- The risk distributions result from all model shocks unrestricted.
- The yield curve based on the local currency interest rate swap computed with the price of risk at 0.6.
- Swap rates of fix to 3M USD Libor used to compute foreign discount factors and long-term rates in construction of the yield curve based on cross-currency fix-to-fix swap.

Yield Curves, Rwanda baseline



Demand Side

IS curve has backward and forward looking terms, monetary condition index, and foreign demand

$$\hat{y}_t = a_1\hat{y}_{t-1} + a_2 E[\hat{y}]_t + a_3 rmci_{t-1} + a_4(a_5\hat{y}^{*EZ}_{t-1} + (1-a_5)\hat{y}^{*EZ}_{t-1}) + \varepsilon^{\hat{y}}_t$$

Real monetary condition index is a combination of real interest rate gap and real exchange rate gap

$$rmci_t = a_6(-\hat{z}_t) + (1-a_6)\hat{r}_t$$

Back to presentation

Supply Side

CPI inflation is a combination of food and ex-food inflations

$$\pi_t = c_5 \pi_t^f + (1 - c_5) \pi_t^{xf}$$

Ex-food Phillips curve:

$$\begin{aligned}\pi_t^{xf} =& b_1 E[\pi^{xf}]_t + (1 - b_1 - b_2 - b_3) \pi_{t-1}^{4xf} + b_2 \pi_{t-2}^{*oil,im} \\ &+ b_3 \pi_t^{*im} + b_4 rmc_t^{xf} + \varepsilon_t^{\pi^{xfood}}\end{aligned}$$

Food Phillips curve:

$$\begin{aligned}\pi_t^f =& b_5 E[\pi^f]_t + (1 - b_5 - b_6 - b_7) \pi_{t-1}^{4f} + b_6 \pi_{t-1}^{*food,im} \\ &+ b_7 \pi_t^{*im} + b_8 rmc_t^f + \varepsilon_t^{\pi^{food}}\end{aligned}$$

Back to presentation

Supply Side (cont.)

Real marginal costs in ex-food production:

$$rmc_t^{xf} = c_3 \hat{y}_t + (1 - c_3 - c_4)\hat{z}_t + c_4 \hat{rp}_t^{oil}$$

Real marginal costs in food production:

$$rmc_t^{f} = c_1 \hat{y}_t + (1 - c_1 - c_2)\hat{z}_t + c_2 \hat{rp}_t^{food}$$

Back to presentation

Monetary Policy

Policy rule:

$$i_t = d_1 i_{t-1} + (1 - d_1)(E[\pi^{tar}]_t + \bar{r}_t + d_2 \hat{\pi}_t + d_3 \hat{y}_t + d_4 \hat{s}_t^{RWF/USD}) + \varepsilon_t^i$$

Inflation forecast deviation from the target:

$$\hat{\pi}_t = (\pi_t^4 + \pi_{t+1}^4 + \pi_{t+2}^4 + \pi_{t+3}^4)/4 - E[\pi^{tar}]_t$$

Exchange rate depreciation rate deviation from its implied short-term target:

$$\hat{s}_t^{RWF/USD} = \Delta s_t^{RWF/USD} - \Delta \bar{z}_t + a_5 \pi_{ss}^{US} + (1 - a_5)(\pi_{ss}^{EZ} + \Delta s_{ss}^{USD/EUR}) - \pi_t^{tar}$$

Back to presentation

Uncovered Interest Rate Parity Condition

Short-term UIP:

$$s_t^{RWF/USD} = E[s^{RWF/USD}]_t - (i_t - i_t^{*US} - prem_t)/4 + \varepsilon_t^{s^{RWF/USD}}$$

Long-term UIP:

$$\begin{aligned}\bar{r}_t =& \bar{r}_t^{*US} + \Delta\bar{z}_t \\ &+ prem_t - (\pi_{ss}^{EZ} - \pi_{ss}^{US} + \Delta s_{ss}^{USD/EUR})/4\end{aligned}$$

Exchange Rate Expectations partly backward looking:

$$\begin{aligned}E[s^{RWF/USD}]_t =& g_2 s_{t+1}^{RWF/USD} + (1-g_2)(s_{t-1}^{RWF/USD} \\ &+ 2/4(\Delta\bar{z}_t + \pi_t^{tar} \\ &- a_5\pi_{ss}^{US} - (1-a_5)(\Delta s_{ss}^{USD/EUR} + \pi_{ss}^{EZ})))\end{aligned}$$

Back to presentation

Expectations

$$
\begin{aligned}
E[\pi^{f}]_t &= \pi^{f}_{t+1} \\
E[\pi^{xf}]_t &= (\pi_{t+1} - c_5\pi^{f}_{t+1})/(1-c_5) \\
E[\hat{y}]_t &= \hat{y}_{t+1} \\
E[\pi 4]_t &= g_1\pi^{4}_{t+1} + (1-g_1)\pi^{4}_{t-1} \\
E[\pi^{tar}]_t &= (\pi^{tar}_{t} + \pi^{tar}_{t+1} + \pi^{tar}_{t+2} + \pi^{tar}_{t+3})/4
\end{aligned}
$$

Back to presentation

Food Prices

$$\pi_t^{*food,im} = \pi_t^{*food} + \Delta s_t^{RWF/USD} - \Delta \bar{z}_t - \Delta \bar{rp}_t^{food}$$

$$rp_t^{food} = p_t^{food} - p_t^{*}$$

$$rp_t^{food} = \bar{rp}_t^{food} + \hat{rp}_t^{food}$$

$$\Delta \bar{rp}_t^{food} = j_7 \Delta \bar{rp}_{t-1}^{food} + \varepsilon_t^{\Delta \bar{rp}^{food}}$$

$$\hat{rp}_t^{food} = j_9 \hat{rp}_{t-1}^{food} + \varepsilon_t^{\hat{rp}^{food}}$$

$$\pi_t^{*food} = 4(p_t^{food} - p_{t-1}^{food})$$

$$\Delta \bar{rp}_t^{food} = 4(\bar{rp}_t^{food} - \bar{rp}_{t-1}^{food})$$

Back to presentation

Oil Prices

$$\pi_t^{*oil,im} = \pi_t^{*oil} + \Delta s_t^{RWF/USD} - \Delta \bar{z}_t - \pi^{*\bar{q}oil}{}_t$$

$$rp_t^{oil} = p_t^{oil} - p_t^*$$

$$rp_t^{oil} = \bar{rp}_t^{oil} + \hat{rp}_t^{oil}$$

$$\pi^{*\bar{q}oil}{}_t = j_8 \pi^{*\bar{q}oil}{}_{t-1} + (1 - j_8)\Delta \bar{rp}_{ss}^{oil} + \varepsilon_t^{\Delta \bar{rp}^{oil}}$$

$$\hat{rp}_t^{oil} = j_{10} \hat{rp}_{t-1}^{oil} + \varepsilon_t^{\hat{rp}^{oil}}$$

$$\pi_t^{*oil} = 4(p_t^{oil} - p_{t-1}^{oil})$$

$$\pi^{*\bar{q}oil}{}_t = 4(\bar{rp}_t^{oil} - \bar{rp}_{t-1}^{oil})$$

Back to presentation

Exogenous Processes for Trends

$$
\begin{aligned}
\Delta \bar{\bar{z}}_t &= h_2 \Delta \bar{\bar{z}}_{t-1} + (1 - h_2) \Delta \bar{\bar{z}}_{ss} + \varepsilon_t^{\Delta \bar{\bar{z}}} \\
\Delta \bar{y}_t &= h_1 \Delta \bar{y}_{t-1} + (1 - h_1) \Delta \bar{y}_{ss} + \varepsilon_t^{\Delta \bar{y}} \\
\Delta \bar{\bar{z}}_t &= 4(\bar{\bar{z}}_t - \bar{\bar{z}}_{t-1}) \\
\Delta \bar{y}_t &= 4(\bar{y}_t - \bar{y}_{t-1}) \\
prem_t &= h_3 prem_{t-1} + (1 - h_3) \bar{r}_{ss} +_t + \varepsilon_t^{prem}
\end{aligned}
$$

Back to presentation

Exogenous Variables

$$\pi_t^{*US} = 4(p_t^{*US} - p_{t-1}^{*US})$$
$$\pi_t^{*EZ} = 4(p_t^{*EZ} - p_{t-1}^{*EZ})$$
$$\pi_t^{*US} = j_1 \pi_{t-1}^{*US} + (1 - j_1)\pi_{ss}^{US} + \varepsilon_t^{\pi^{US}}$$
$$\pi_t^{*EZ} = j_2 \pi_{t-1}^{*EZ} + (1 - j_2)\pi_{ss}^{EZ} + \varepsilon_t^{\pi^{EZ}}$$
$$\pi_t^{*4US} = (\pi_t^{*US} + \pi_{t-1}^{*US} + \pi_{t-2}^{*US} + \pi_{t-3}^{*US})/4$$
$$\pi_t^{*4EZ} = (\pi_t^{*EZ} + \pi_{t-1}^{*EZ} + \pi_{t-2}^{*EZ} + \pi_{t-3}^{*EZ})/4$$
$$\pi_t^{*im} = \pi_t^{*} + \Delta s_t^{RWF/USD} - \Delta \bar{z}_t$$

Back to presentation

Exogenous Variables (cont.)

$$
\begin{aligned}
p_t^* &= a_5 p_t^{*US} + (1 - a_5)(p_t^{*EZ} + s_t^{USD/EUR}) \\
\pi_t^* &= 4(p_t^* - p_{t-1}^*) \\
\pi_t^{tar} &= h_5 \pi_{t-1}^{tar} + (1 - h_5)\pi_{ss}^{tar} + \varepsilon_t^{\pi^{tar}} \\
\bar{r}_t^{*US} &= j_6 \bar{r}_{t-1}^{*US} + (1 - j_6)\bar{r}_{ss}^{US} + \varepsilon_t^{\bar{r}^{US}} \\
i_t^{*US} &= j_3 i_{t-1}^{*US} + (1 - j_3)(\bar{r}_{ss}^{US} + \pi_{ss}^{US}) + \varepsilon_t^{i^{US}} \\
\Delta s_t^{USD/EUR} &= \Delta s_{ss}^{USD/EUR} + \varepsilon_t^{\Delta s^{USD/EUR}} \\
s_t^{USD/EUR} &= s_{t-1}^{USD/EUR} + \Delta s_t^{USD/EUR}/4 \\
\hat{y}_t^{*EZ} &= j_5 \hat{y}_{t-1}^{*EZ} + \varepsilon_t^{\hat{y}^{EZ}} \\
\hat{y}_t^{*EZ} &= j_4 \hat{y}_{t-1}^{*EZ} + \varepsilon_t^{\hat{y}^{US}}
\end{aligned}
$$

Real Identities

$$
\begin{aligned}
r_t &= i_t - E[\pi 4]_t \\
r_t &= \bar{r}_t + \hat{r}_t \\
\Delta z_t &= 4(z_t - z_{t-1}) \\
z_t &= s_t^{RWF/USD} - p_t + p_t^* \\
z_t &= \bar{z}_t + \hat{z}_t \\
\Delta y_t &= 4(y_t - y_{t-1}) \\
y_t &= \bar{y}_t + \hat{y}_t \\
y_t^{year} &= (y_t + y_{t-1} + y_{t-2} + y_{t-3})/4 \\
\Delta 4 y_t &= (\Delta y_t + \Delta y_{t-1} + \Delta y_{t-2} + \Delta y_{t-3})/4
\end{aligned}
$$

Back to presentation

Nominal Identities

$$
\begin{aligned}
\pi_t^4 &= (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})/4 \\
\pi_t^{4f} &= (\pi_t^f + \pi_{t-1}^f + \pi_{t-2}^f + \pi_{t-3}^f)/4 \\
\pi_t^{4xf} &= (\pi_t^{xf} + \pi_{t-1}^{xf} + \pi_{t-2}^{xf} + \pi_{t-3}^{xf})/4 \\
\pi_t &= 4(p_t - p_{t-1}) \\
\pi_t^f &= 4(p_t^f - p_{t-1}^f) \\
\pi_t^{xf} &= 4(p_t^{xf} - p_{t-1}^{xf})
\end{aligned}
$$

Back to presentation

Compounded Interest Rates

$$
\begin{aligned}
i_t^4 &= (i_t + i_{t-1} + i_{t-2} + i_{t-3})/4 \\
i_t^8 &= (i_t^4 + i_{t-4}^4)/2 \\
i_t^{12} &= (2i_t^8 + i_{t-8}^4)/3 \\
i_t^{16} &= (3i_t^{12} + i_{t-12}^4)/4 \\
i_t^{4*US} &= (i_t^{*US} + i_{t-1}^{*US} + i_{t-2}^{*US} + i_{t-3}^{*US})/4 \\
i_t^{8*US} &= (i_t^{4*US} + i_{t-4}^{4*US})/2 \\
i_t^{12*US} &= (2i_t^{8*US} + i_{t-8}^{4*US})/3 \\
i_t^{16*US} &= (3i_t^{12*US} + i_{t-12}^{4*US})/4
\end{aligned}
$$

Back to presentation

Excess Return

$$er_t^4 = s_{t-4}^{RWF/USD} - s_t^{RWF/USD} + i_{t-1}^4 - i_{t-1}^{4*US}$$

$$er_t^8 = (s_{t-8}^{RWF/USD} - s_t^{RWF/USD})/2 + i_{t-1}^8 - i_{t-1}^{8*US}$$

$$er_t^{12} = (s_{t-12}^{RWF/USD} - s_t^{RWF/USD})/3 + i_{t-1}^{12} - i_{t-1}^{12*US}$$

$$er_t^{16} = (s_{t-16}^{RWF/USD} - s_t^{RWF/USD})/4 + i_{t-1}^{16} - i_{t-1}^{16*US}$$

Back to presentation