Nonlinear DSGE Model of the Czech Economy with Time-varying Parameters

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Motivation

- Did any structural changes occur during the turbulent period of recent financial and economic crisis of 2008–2009?
- Which structural parameters did change? Were the changes temporary or permanent?
- How was the behaviour of the economy affected by these structural changes?
- Which changes are specific for the Czech economy and which correspond to the europe-wide trends?

Time-varying parameters within state-space models

- time-varying parameters are defined as unobserved states

$$\theta_t = (1 - \alpha_t^\theta) \cdot \theta_{t-1} + \alpha_t^\theta \cdot \overline{\theta} + \nu_t^\theta$$

- $\overline{\theta}$ is initial value of parameter θ_t
- α_t^θ is a time-varying adhesion parameter (panel)
  - $\alpha^\theta = 0 \Rightarrow$ random walk,
  - $\alpha^\theta = 1 \Rightarrow$ white noise around $\overline{\theta}_t$,
  - $\alpha^\theta = 0.25 \Rightarrow$ our choice
- $\nu_t^\theta \sim N(0, \sigma_\nu^\theta)$

$\Rightarrow$ nonlinearity is introduced into the model $\Rightarrow$ nonlinear state-space model

$$x_t = g(x_{t-1}, w_{t-1})$$

$$y_t = h(x_t, v_t)$$

Non-linear filtering methods

- Kalman filter is optimal for linear systems
- Extended Kalman filter (Jacobian matrix of the state vector) can be used for nonlinear systems but performs poorly for severe nonlinearities

$\Rightarrow$ Nonlinear filters
  - with additive Gaussian noise - Extended Kalman filters
    - Monte Carlo based
    - Transformation based
  - with non Gaussian noise - Particle filters
    - Gaussian particle filter
    - Unscented particle filter

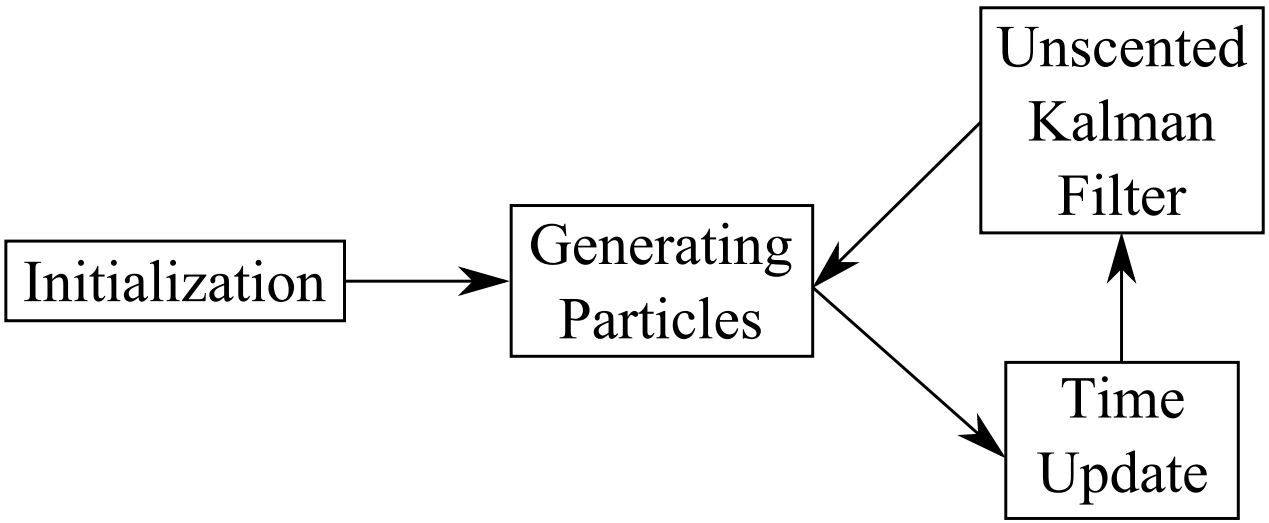
Non-linear particle filter

1. Initialization: $t = 0$, set the prior mean $\overline{x}_0$ and covariance matrix P_0 for the state vector x_t.
2. Generating particles: Draw a total of N particles $x_{t+1}^{(i)}$, $i = 1, \ldots, N$ from distribution $p(x_{t+1})$ with mean $\overline{x}_{t+1|t}$ and covariance matrix $P_{t+1|t}$ (transition equation). Calculate $\overline{y}_{t+1|t}$ (measurement equation) and covariance matrices $P_{y,y}$ and $P_{x,y}$.
3. Kalman filter:

$$K_{t+1} = P_{x,y} \left(P_{y,y}\right)^{-1},$$
$$\overline{x}_{t+1} = \overline{x}_{t+1|t} + K_{t+1}(y_{t+1} - \overline{y}_{t+1|t}),$$
$$P_{t+1} = P_{t+1|t} - K_{t+1} P_{y,y} \left(K_t\right)^T$$

4. Time Update: $t = t + 1$.

Non-linear particle filter, diagram



Non-linear particle filter setting

- 20 runs of the UPF with 30.000 particles each were calculated for the second order approximation of the model.
- Initial values of the time-varying parameters ($\overline{\theta}$) were set to the posterior means of the Bayesian estimation of the model with constant parameters
- Standard deviations of time-varying parameter innovations (σ_{ν}^{θ}) were set proportional to the estimated posterior means (10 %).
- Bayesian Random Walk Metropolis-Hastings estimation: two chains of 1.000.000 draws each, 50% burn-in sample, acceptance rate near 30%.

Model

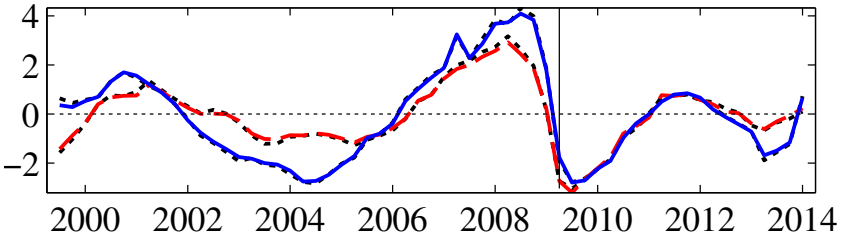
- Overall structure of the DSGE model of a small open economy (SOE) is based on Shaari (2008), who incorporated the financial accelerator mechanism á la Bernanke *et al.* (1999) into the basic SOE model of Galí and Monacelli (2005).
- The model contains following optimizing representative agents: households, entrepreneurs and domestic and foreign retailers.
- The monetary policy of the central bank is modelled with the use of forward looking Taylor rule.
- Foreign sector observables are modelled as SVAR(1) block.

Data

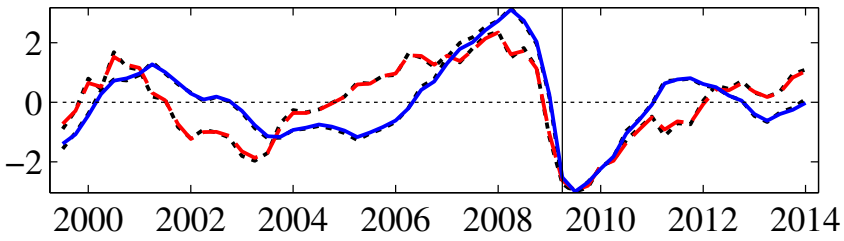
- The model is estimated on two sets of data: CZ+EA (blue) and EA+US (red)
- Quarterly time series of the period between 1999Q2 and 2013Q4, 59 observations
- Domestic economy: real aggregate product, real investment, consumer price index , 3-month PRIBOR (3-month EURIBOR)
- Foreign economy EA17 (US): real aggregate product, CPI index and 3-month EURIBOR (3-month T-Bill rate)
- CZK/EUR (EUR/USD) real exchange rate
- Original time series were transformed so as to express percentage deviations from steady state (HP filter, $\lambda = 1600$)

Filtered observables (deviations from steady state in per cent)

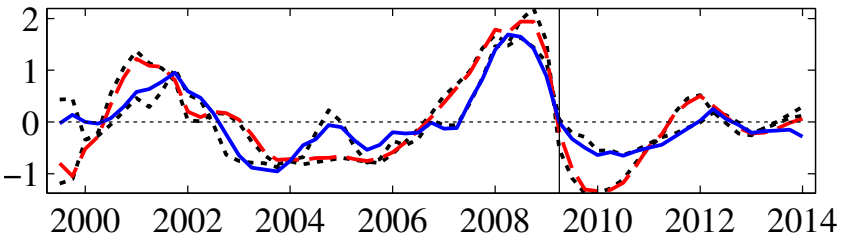
y_{obs} ... observed real output



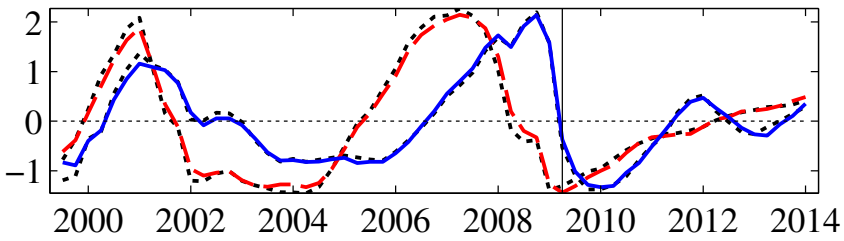
y^*_{obs} ... observed foreign real output



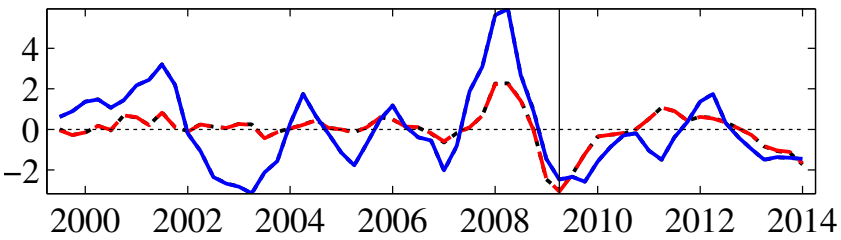
r_{obs} ... observed interest rate



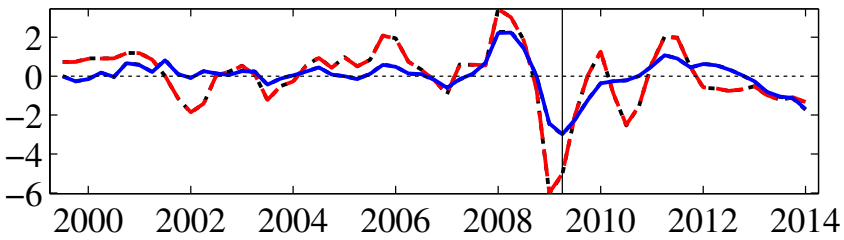
r^*_{obs} ... observed foreign interest rate



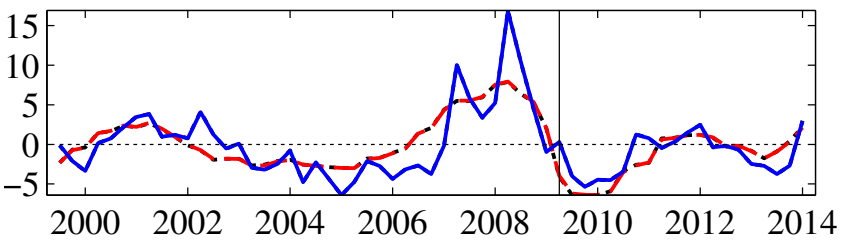
π_{obs} ... observed CPI inflation



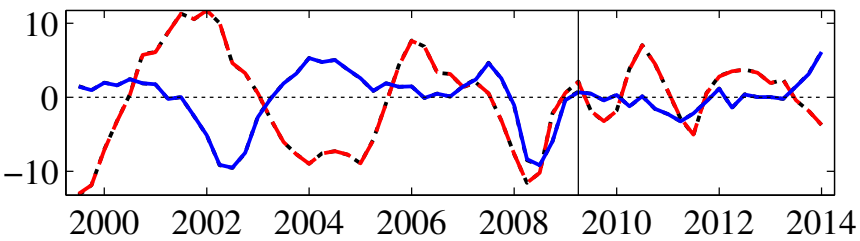
π^*_{obs} ... observed foreign CPI inflation



inv_{obs} ... observed real investment



rer_{obs} ... observed real exchange rate



Calibration

Parameter		Value
β	Discount factor	0.995
α	Capital share in production	0.350
δ	Capital depreciation rate	0.025
μ	Steady-state domestic mark-up	1.200
Ω	Household's share in labour supply	0.990

Estimation results

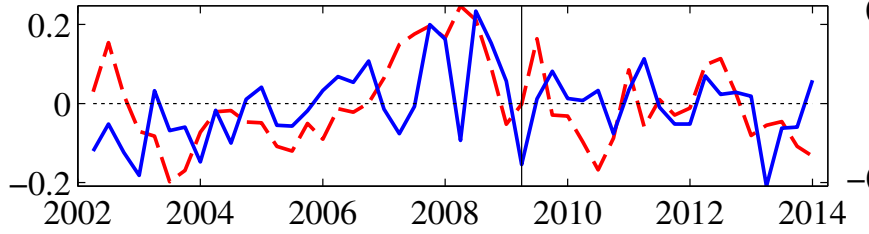
Parameter		Distribution	Prior Mean	Prior Std	CZ Posterior Mean	CZ Posterior Std	EA Posterior Mean	EA Posterior Std
Structural parameters								
Υ	Habit persistence	B	0.60	0.05	0.60	0.05	0.68	0.06
Ψ	Inv. elast. of lab. supply	G	2.00	0.50	1.25	0.35	0.88	0.26
ψ^B	Debt-elastic risk premium	G	0.05	0.02	0.02	0.01	0.02	0.01
η	Home/foreign elast. subst.	G	0.65	0.10	0.52	0.08	0.43	0.02
κ	Price indexation	B	0.50	0.10	0.49	0.09	0.44	0.09
γ	Pref. bias to foreign goods	B	0.40	0.15	0.48	0.07	0.27	0.04
θ_H	Home goods Calvo	B	0.70	0.10	0.82	0.03	0.80	0.03
θ_F	Foreign goods Calvo	B	0.70	0.10	0.84	0.02	0.81	0.03
ψ^I	Capital adjustment costs	G	8.00	3.00	11.5	2.92	15.5	3.35
Financial frictions								
Γ	Leverage ratio ss ratio	G	2.00	0.50	1.41	0.24	1.16	0.21
ς	Bankruptcy rate	B	0.025	0.015	0.05	0.02	0.02	0.01
χ	Financial accelerator	G	0.05	0.015	0.04	0.01	0.04	0.01
Taylor rule								
ρ	Interest rate smoothing	B	0.70	0.10	0.86	0.02	0.74	0.04
β_π	Inflation weight	G	1.50	0.20	1.75	0.23	1.76	0.23
Θ_y	Output gap weight	G	0.50	0.20	0.16	0.05	0.22	0.06

Estimation results, shock parameters

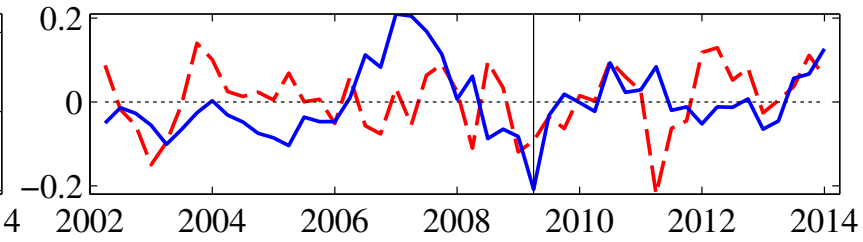
Parameter		Distribution	Prior Mean	Prior Std	Posterior Mean CZ	Posterior Mean EA
Autocorrealtion coefficients						
ρ_Y	Domestic productivity	B	0.50	0.20	0.58	0.47
ρ_{UIP}	Uncovered interest parity	B	0.50	0.20	0.63	0.79
ρ_{LOP}	Law of one price	B	0.50	0.20	0.93	0.84
ρ_{NW}	Entrepreneurial net worth	B	0.50	0.20	0.44	0.40
Standard deviations						
σ_Y	Domestic productivity	IG	1.00	∞	1.09	0.38
σ_{UIP}	Uncovered interest parity	IG	0.50	∞	0.27	0.24
σ_{LOP}	Law of one price	IG	0.50	∞	3.22	5.05
σ_{NW}	Entrepreneurial net worth	IG	1.00	∞	1.84	1.48
σ_{MP}	Monetary policy	IG	0.50	∞	0.08	0.10
σ_{y^*}	Foreign output	IG	1.00	∞	0.52	0.54
σ_{π^*}	Foreign inflation	IG	0.50	∞	0.14	0.30
σ_{r^*}	Foreign interest rate	IG	0.50	∞	0.08	0.10

Measurement errors (deviations from steady state in per cent)

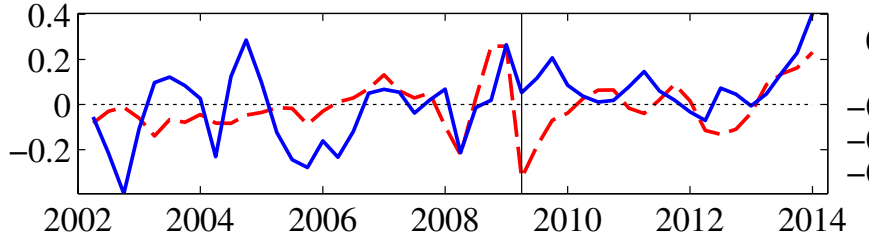
$y_{obs} - y$ ... real output error



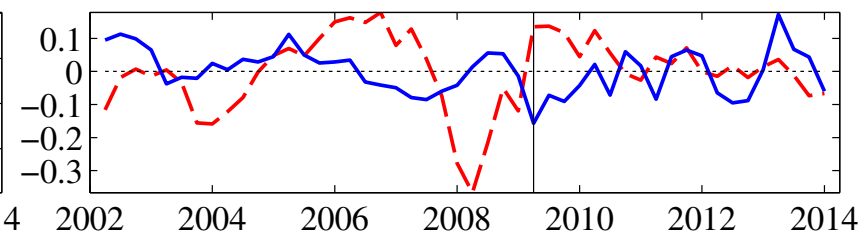
$y^*_{obs} - y^*$ ... foreign real output error



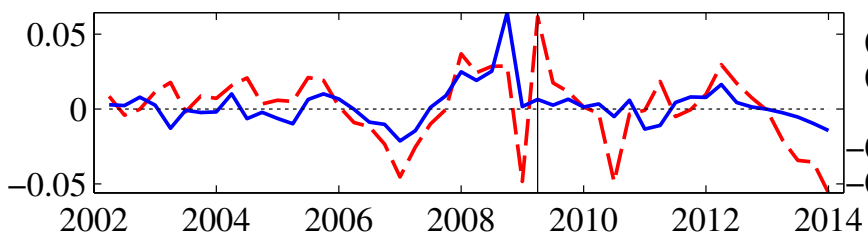
$r_{obs} - r$ ... interest rate error



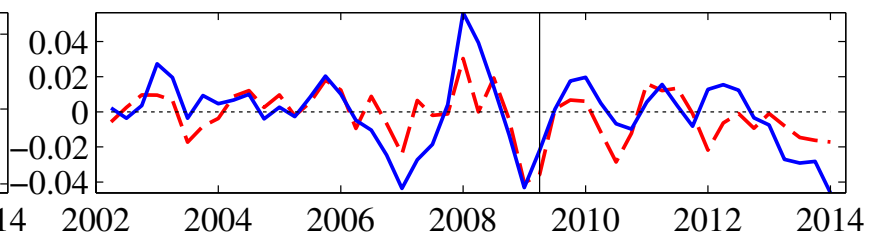
$r^*_{obs} - r^*$ ... foreign interest rate error



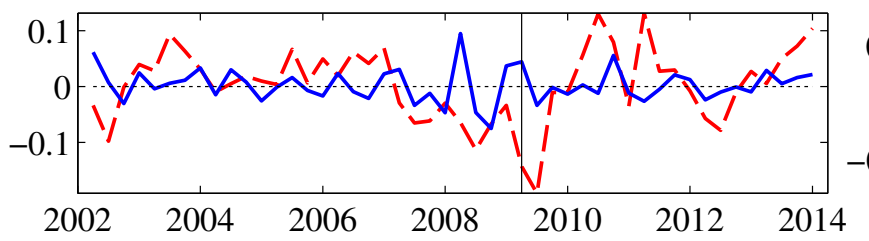
$\pi_{obs} - \pi$ ... CPI inflation error



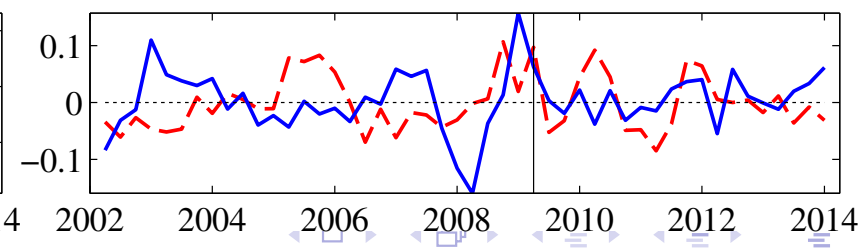
$\pi^*_{obs} - \pi^*$ ... foreign CPI inflation error



$inv_{obs} - inv$ ... real investment error

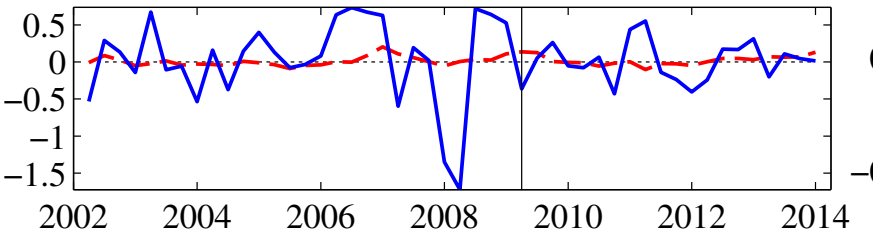


$rer_{obs} - rer$ ... real exchange rate error

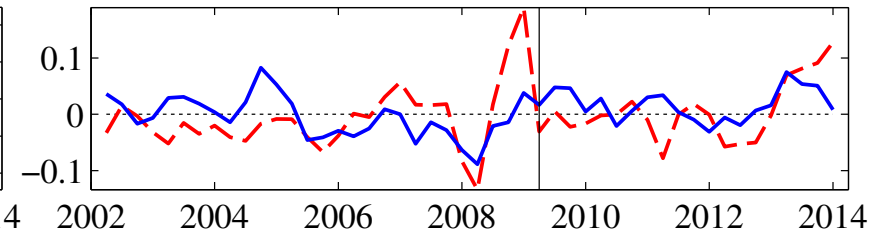


Filtered shock innovations (deviations from steady state in per cent)

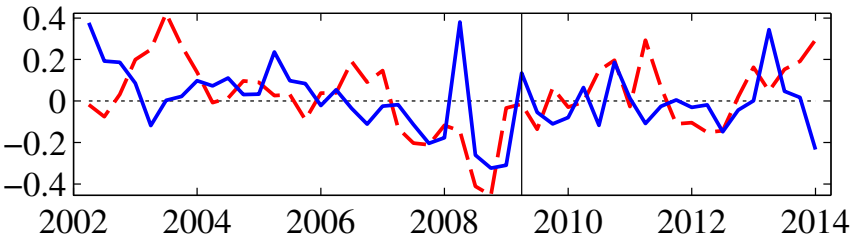
ε_y ... productivity innovations



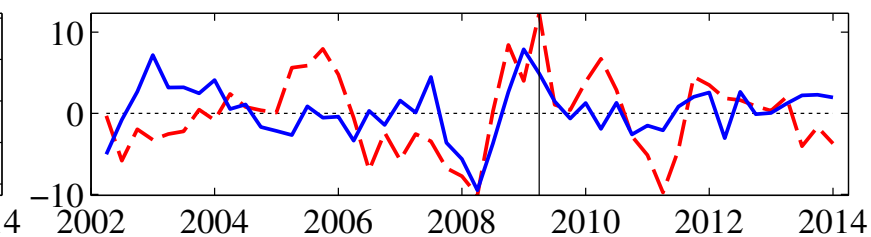
ε_{MP} ... monetary policy innovations



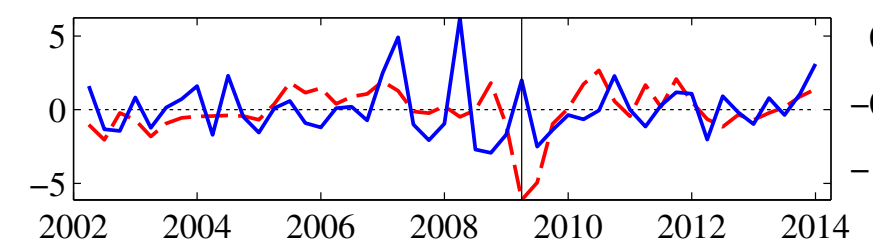
ε_{UIP} ... UIP innovations



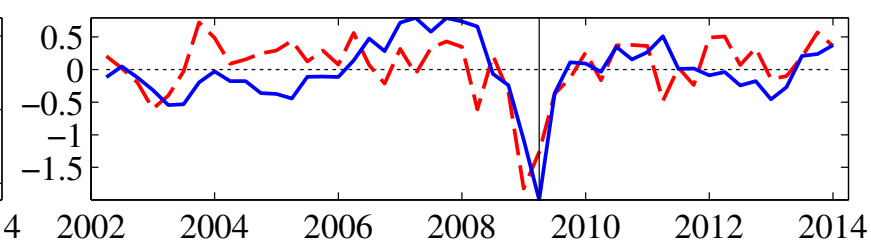
ε_{LOP} ... LOP innovations



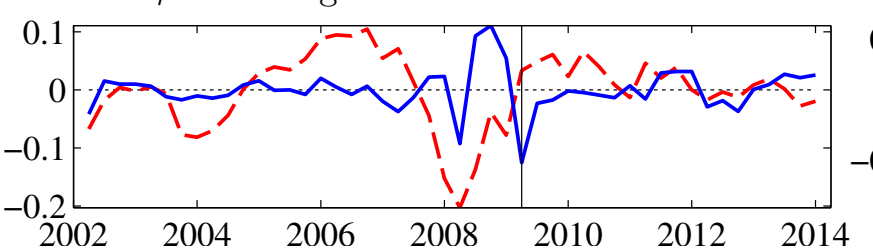
ε_{NW} ... survival rate innovations



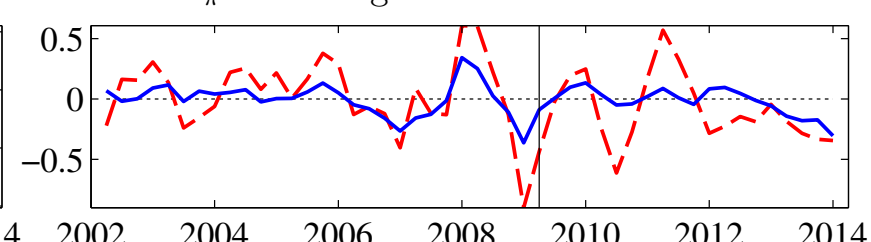
ε_{y^*} ... foreign output innovations



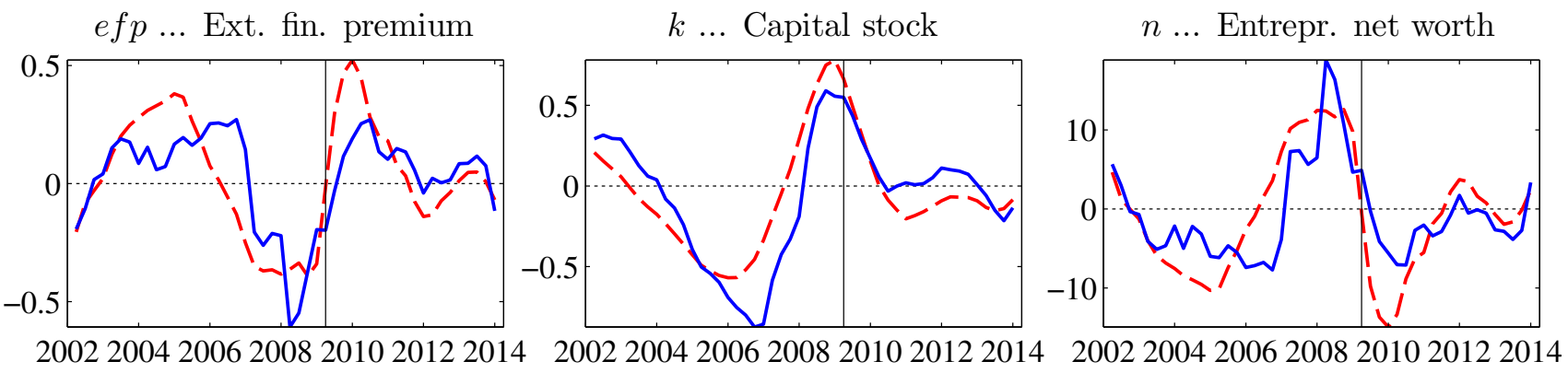
ε_{r^*} ... foreign interest rate innovations



ε_{π^*} ... foreign inflation innovations

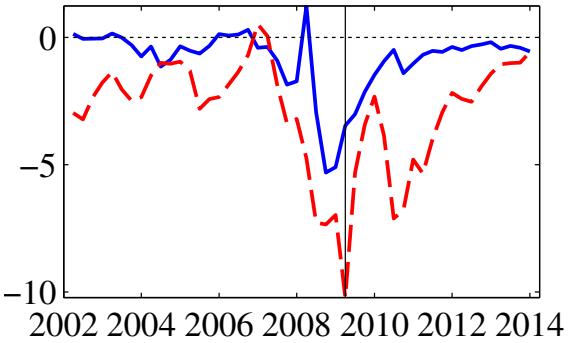


Selected filtered variables (deviations from steady state in per cent)

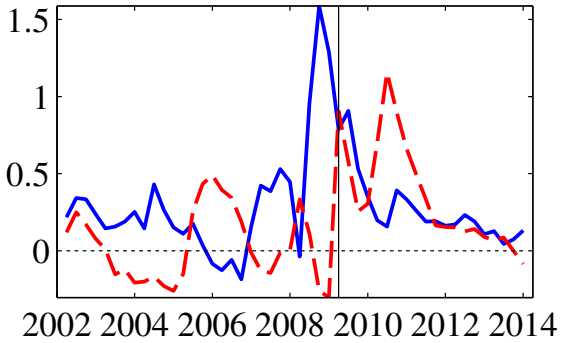


Time-varying parameters (deviations from initial value in per cent)

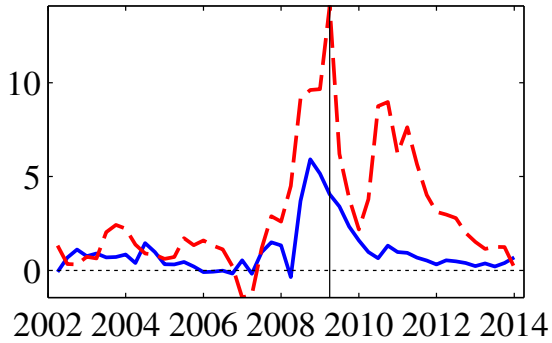
χ ... financial accelerator



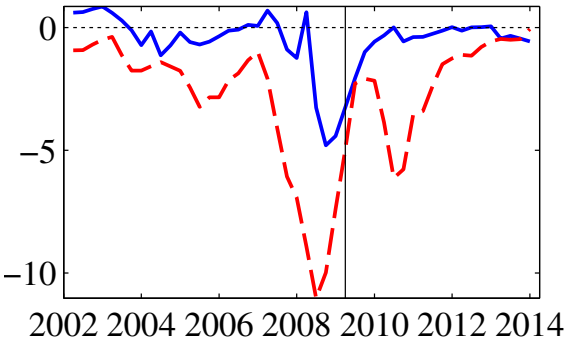
ς ... bankruptcy rate



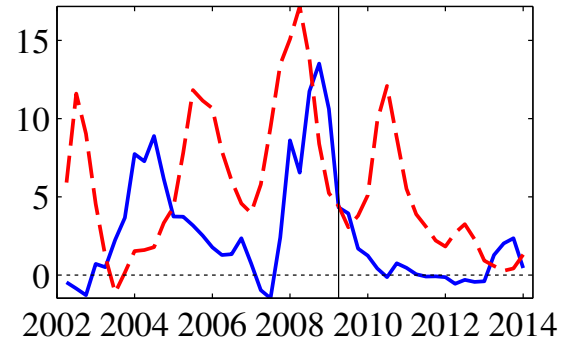
Ψ_I ... capital adj. costs



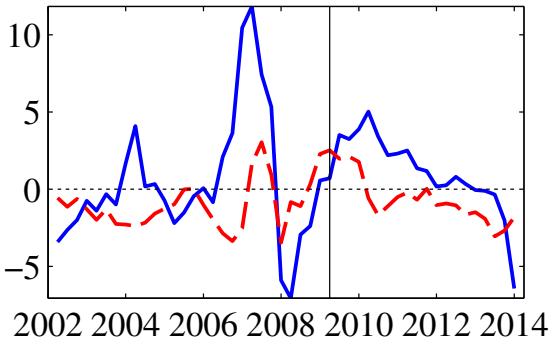
Γ ... leverage rate



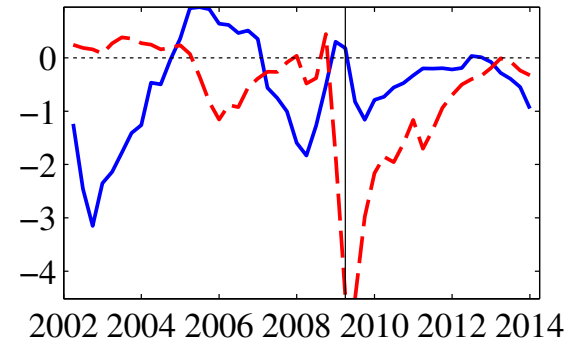
γ ... foreign goods preference bias



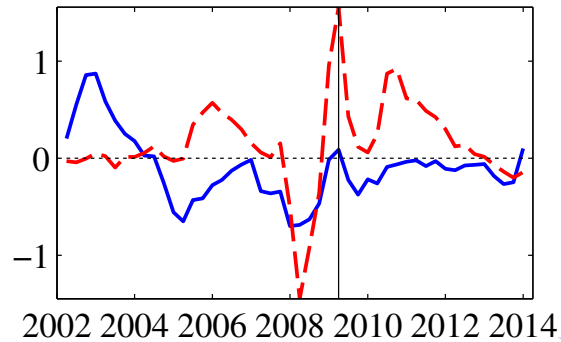
θ_H ... domestic Calvo param.



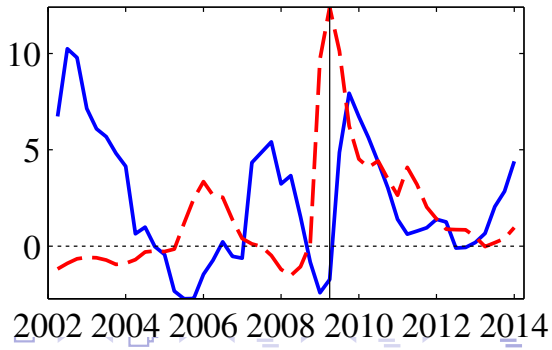
β_π ... Taylor rule, inflation



Θ_y ... Taylor rule, output gap



ρ ... Taylor rule, smoothing



Correlation of time-varying parameters

Parameter		Correlation
χ	Financial accelerator	0.66
ς	Bankruptcy rate	-0.13
Ψ_I	Capital adj. costs	0.71
Γ	Leverage ratio ss	0.64
γ	Foreign goods pref. bias	0.19
θ_H	Domestic Calvo param.	0.34
β_π	Taylor rule, inflation	-0.20
Θ_y	Taylor rule, output gap	0.25
ρ	Tylor rule, smoothing	-0.18

Conclusion

- Results of the estimation suggest that some structural changes occurred during recent financial and economic crisis
- The structural changes were probably temporary as the parameters tend to return to their initial values
- Some parameters showed only negligible deviations from their initial values (elast. of intertemp subst., risk premium elast., inflation indexation)
- Some parameters of the financial sector, openness parameter, Calvo parameters and interest rate smoothing parameter changed markedly during the recent economic crisis

Conclusion (continued)

- Overall, the estimated trajectories show many similarities between the development in the Czech economy and in the euro area with some differences in the magnitude of the deviations and timing.
- The differences can be attributed to earlier onset and more dramatic course of the financial crisis in the euro area than in relatively sheltered Czech economy.
- The trajectories of the Taylor rule parameters also show interesting differences in the behaviour of the ECB and CNB.

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Thank you for your attention!

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