Inside Information and Optimal Exercise of Employee Stock Options

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Modern Tools for Financial Analysis and Modeling
7th June 2016

Outline

1) Employee stock options and inside information

2) Modelling framework

3) Optimal ESO exercises

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An employee stock option (ESO):

- An American call option on the firm's stock granted by a firm to its employees as a benefit in addition to the salary.
- The typical ESO payoff is $\max(S_t - K, 0)$, where S_t it the firm's stock price at time t, and K is the option strike.
- Popular in the US: 94% of companies in the S&P 500 grant ESOs to their top executives.
- ESO holders are not allowed to sell ESOs.
- ESOs are long-term options with maturity up to 10 years.

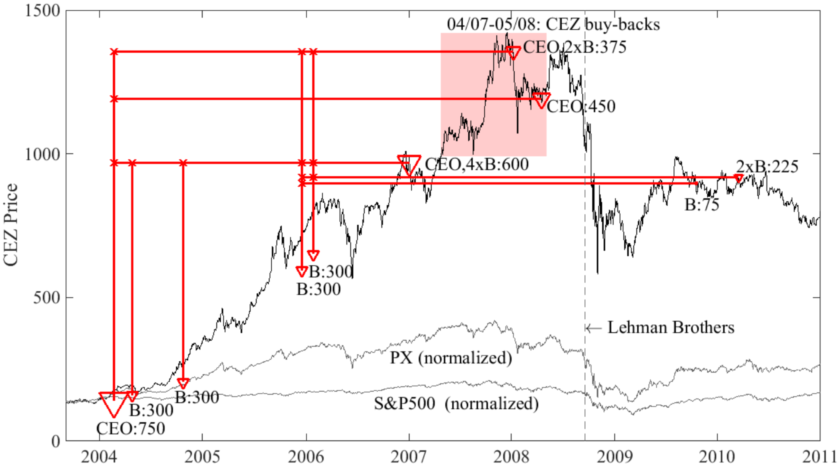
ČEZ ESOs

- ČEZ is the largest electricity producer in the Czech Republic.
- ČEZ is from 70% state owned.
- ČEZ introduced a controversial ESO program for its top executives in 2001.
- The ČEZ ESO was an American call option granted at the money with 5-year maturity.
- Between 2004–06 the ČEZ CEO and 4 board members were granted 1 950 000 ESOs in total.
- The ČEZ CEO cashed in 800 000 000 CZK during his 4-year tenure.

ČEZ on the stock market

- The average annual stock price growth during 01/2004–01/08 was an incredible 175%.
- ČEZ pays out regularly high dividends (dividend yield of 5–8%).
- ČEZ bought back 10% of its own shares from the market between 04/2007–05/08. ČEZ gave two reasons for the buyback:
  1. Reduction of the equity capital of the company.
  2. Fulfilling the commitments arising from the joint-stock option programs namely within the amount of 5 million pieces.

ESO exercises of the ČEZ top executives



ESO exercises: luck or inside information?

- The ČEZ top executives were able to exercise their ESOs at the very profitable moments:
- *Were the ČEZ top executives just lucky?*
- The top executives have more current and detailed information about the company than regular agents, which raises another question:
- *Did ČEZ top executives use this extra or inside information?*

- The previous two questions seem to be unanswerable from the available data set on ČEZ ESO exercises.
- Our goal is to construct a model, which differentiates between the *regular* and *inside* information about the stock price.
- We then use the model to examine the optimal exercise behaviour under the both information cases.
- The exploitation of inside information in the ESO exercises has been studied empirically for larger data sets.
- However, to our knowledge only one theoretical model has been suggested to consider ESO exercises and inside information.

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Inside information by random drift switching

- To be able to generate ESO exercises at the local maxima of the stock price, we allow the stock price drift to switch from a positive to negative value.
- The drift switch is driven by a first jump of a Poisson process independent of the stock price process.
- The differential between the regular and inside information is introduced by observability of the drift switch:
  - The top executive observes the drift switch, and thus has *full information*. We refer to the top executive as the *insider*.
  - The regular agent does not observe the drift switch, and thus needs to filter the drift value from the stock price process. The regular agent has *partial information*, and we call her or him the *outsider*.

The stock price process

- On a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ there exist a standard Brownian motion W_t and a continuous-time Markov chain N_t with two states 0 and 1.
- The stock price process follows the modulated geometric Brownian motion

$$dS_t = \big(\mu_0(1 - N_t) + \mu_1 N_t\big) S_t dt + \sigma S_t dW_t, \quad S_0 = s, \ N_0 = 0, \tag{1}$$

  with the Markov chain N_t given by

$$N_t = \mathbf{1}_{\{t \geq \eta\}}, \tag{2}$$

  where η is an exponentially distributed random time with intensity λ.
- The instantaneous expected stock return switches at the random time η from a constant value μ_0 to a constant value μ_1, whereas the return volatility σ remains constant.
- We assume that $\mu_0 > \mu_1$.

Optimal ESO exercise problem

- The ESO is written at time 0 and expires at time T.
- The ESO holder would like to optimally exercise their ESO written on the stock with price dynamics (1).
- In particular, the ESO holder maximises their utility of the option payoff and a nonstochastic wealth.
- The ESO holder thus solves an *optimal stopping problem* given by

$$\underset{0 \leq \tau \leq T}{\text{maximise}} \ \mathbb{E}\big[G(\tau, S_\tau)\big], \tag{3}$$

  where $G(t, s)$ is the reward function implied from the ESO payoff and possession of the nonstochastic wealth.
- We solve the optimal stopping problem (3) under the full and partial information by dynamic programming.

- In the full information case, both the stock price process and the drift switching process are observable to the ESO holder.
- The insider has an access to a filtration generated by S_t and N_t, and given by $\mathcal{F}_t^{S,N} = \sigma\big(S_u, N_u \big|\ u \le t\big)$.
- The joint process (S_t, N_t) is Markov with respect to its natural filtration $\mathcal{F}_t^{S,N}$. Given the Markovian structure, the insider's optimal stopping problem at time 0 is given by the value function

$$V_{\mathrm{f}}(0, s, l) = \sup_{0 \le \tau \le T} \mathbb{E}\Big[G(\tau, S_\tau)\Big|S_0 = s, N_0 = l\Big], \quad l \in \{0, 1\}. \tag{4}$$

- In the partial information case, only the stock price process is observed by the ESO holder.
- The outsider's filtration is given by $\mathcal{F}_t^S = \sigma\big(S_u \big|\ u \leq t\big)$, and the outsider uses the Wonham filter to estimate the probability of the drift value.
- The filter leads to the stock price process representation

$$dS_t = (\mu_0(1 - \Pi_t) + \mu_1 \Pi_t) S_t dt + \sigma S_t d\overline{W}_t, \quad S_0 = s, \tag{5}$$

where $\Pi_t = \mathbb{P}(N_t = 1 | \mathcal{F}_t^S)$.
- The filtered probability Π_t solves the stochastic differential equation

$$d\Pi_t = \big(\lambda_0(1 - \Pi_t) - \lambda_1 \Pi_t\big) dt + \frac{\mu_1 - \mu_0}{\sigma} \Pi_t (1 - \Pi_t) d\overline{W}_t, \quad \Pi_0 = \pi, \tag{6}$$

where the innovation process $\overline{W}_t$ is a standard Brownian motion.
- The joint process $(S_, \Pi_t)$ is Markov with respect to its natural filtration $\mathcal{F}_t^S$. Given the Markovian structure, the outsider's optimal stopping problem at time 0 is given by the value function

$$V_{\mathrm{p}}(0, s, \pi) = \sup_{0 \leq \tau \leq T} \mathbb{E}\Big[G(\tau, S_\tau) \Big| S_0 = s, \Pi_0 = \pi\Big]. \tag{7}$$

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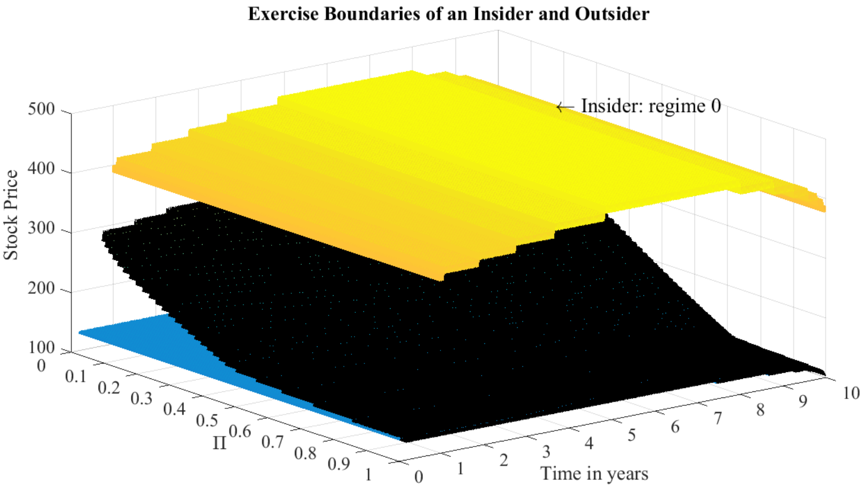
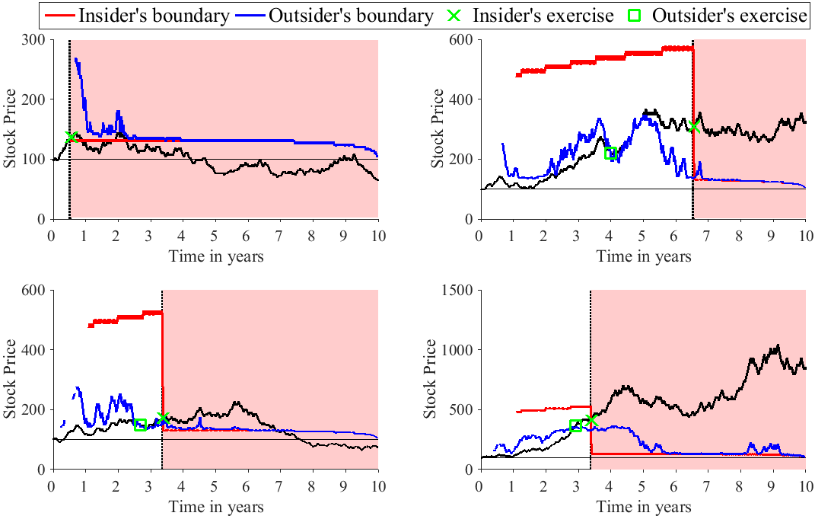
- Numerical solutions, for example, PDE or lattice methods, to the full information case (4) are well studied in the literature.
- The partial information case (7) is a considerably harder problem. To our knowledge, no ready to use numerical methods have been suggested.
- We develop binomial tree algorithms to solve the both cases.

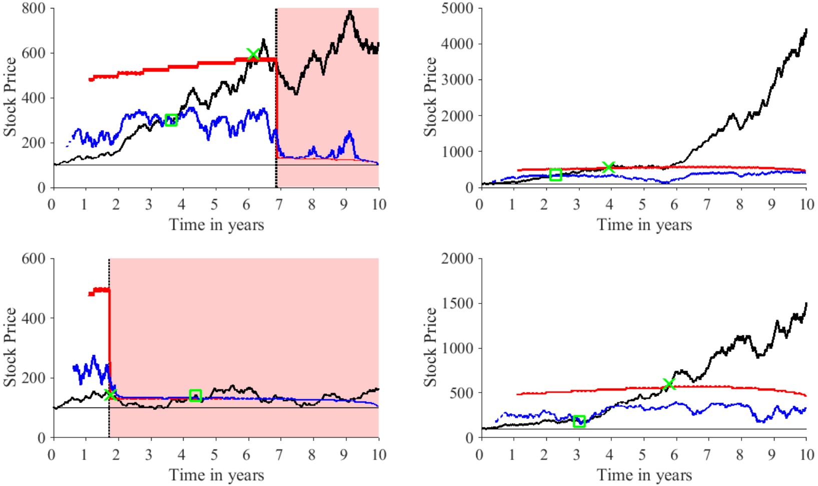
- An American call option with 10-year maturity is granted to the insider and outsider.
- The ESO is granted at the money, $S_0 = K = 100$, and its payoff is $\max(S_t - K, 0)$.
- The economy is parameterised as follows.
  - Stock price drift and volatility: $\mu_0 = 25\%$, $\mu_1 = -1\%$, $\sigma = 20\%$.
  - Risk-free rate: $r = 4\%$.
  - Intensity of the drift switching process N_t: $\lambda = 15\%$. This intensity value implies that the probability of μ_0 switching to μ_1 during the option life is 78%.
- The ESO holder maximises, with respect to the stopping time τ, an exponential utility function with risk aversion parameter $\gamma = 0.03$ and initial nonstochastic wealth $x = 200$:

$$\underset{0 \le \tau \le T}{\text{maximise}}\ \mathbb{E}\left[-\exp\left\{-\gamma \max(S_\tau - K, 0)e^{r(T-\tau)} - \gamma x e^{rT}\right\}\right]. \qquad (8)$$


Exercise Boundaries of an Insider and Outsider
← Insider: regime 0
Stock Price
500
400
300
200
100
Π
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Time in years
0
1
2
3
4
5
6
7
8
9
10

Exercise patterns of the insider and outsider I





Stock Price
Time in years
800
600
400
200
0
0 1 2 3 4 5 6 7 8 9 10
5000
4000
3000
2000
1000
0
0 1 2 3 4 5 6 7 8 9 10
600
400
200
0
0 1 2 3 4 5 6 7 8 9 10
2000
1500
1000
500
0
0 1 2 3 4 5 6 7 8 9 10