STATISTICAL PROCESSING AND NEURAL NETWORK MODELS FOR ECONOMIC TIME SERIES

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Abstract

The paper is devoted to the presentation of methods of economic time series analysis and modelling using the Box-Jenkins methodology, the signal processing approach and the feedforward conventional/fuzzy neural network technique. Some results on our research on time series modelling with emphasis on potential improving forecast accuracy are presented here. The assessment of the particular models has been made using the root mean square error.

1 Introduction

Artificial neural networks (ANN) are now being applied to many problems far removed from their first beginnings. Application to management problems have included predicting bankruptcy [2], predicting ratings of corporate bonds [10], forecasting financial markets [6] and time series forecasting [6]. Their main strengths lie in pattern recognition and have been a hot topic of research for many years now.

There is much controversy about the application of traditional statistical or econometric models and the ANN approaches within the field of economic time series modelling and forecasting. These controversies are based on the assumptions that there is no consensus at all on whether there is chaos in economic time series or not. Various tests for nonlinear pattern and chaos in time series have been proposed to illustrate the nonlinear nature of certain processes. A survey of these tests is presented in [1].

The goal of this paper is to illustrate three areas: probabilistic, adaptive signal processing and computational networks may be used to economic time series modelling. In Section 2, we can see that a random process of time series of stock prices may be generated as the output of linear filter driven by white noise. In Section 3 of this paper, we report on an ANN application that was designed and run by [7], [9] to investigate the problem of forecast accuracy across proposed models

2 Application of the Box-Jenkins methodology in the stock prediction problem

In this section, we give an example that provides one kind of possible results. We will regard these results as the referential values for the approach of adaptive signal processing procedures and ANN modelling. Many of modelling techniques of autoregressive processes are based on recent developments in time series analysis recently consolidated and presented by Box and Jenkins [3].

To illustrate the Box-Jenkins methodology, consider the stock price time readings of a typical company (say VAHOSTAV company). We would like to develop a time series model for this process so that a predictor for the process output can be developed. The data was collected for the period January 2, 1997 to December 31, 1997 which provided a total of 163 observations (see Fig. 1).

To build a forecast model the sample period for analysis y_1, ..., y_{128} was defined, i.e. the period over which the forecasting model was developed and the ex post forecast period (validation data set), y_{129}, ..., y_{163} as the time period from the first observation after the end of the sample period to the most recent observation. By using only the actual and forecast values within the ex post forecasting period only, the accuracy of the model can be calculated.

To determine appropriate Box-Jenkins model, a tentative ARMA model in identification step is identified. In Figure 2, the estimate of autocorrelation $(\hat{r}_k)$ and partial autocorrelation $\hat{a}_{kk}$ function (ACF, PACF) of the data are given. To test whether the autocorrelation and partial autocorrelation coefficient are statistically equal to zero, we use the *t*-statistic $t_r = \hat{r}_k \, / \, S(\hat{r}_k)$ and $t_a = \hat{a}_{kk} \, / \, S(\hat{a}_{kk})$ where

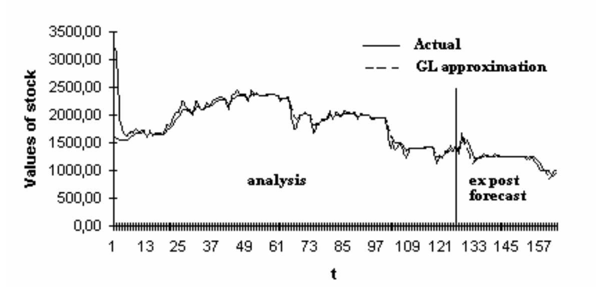

Figure 1: The data for VAHOSTAV stock prices (January 1997 - December 1997) and the values of the AR(7) model for VAHOSTAV stock prices estimated by GL algorithm

$$S(\hat{r}_k) = N^{-1/2}\left[1 + 2\sum_{j=1}^{k-1}\hat{r}_j\right]$$

and

$$S(\hat{a}_{kk}) = N^{-1/2}$$

denote standard errors of the *k*-th sample autocorrelation or partial autocorrelation coefficient, respectively, *N* is the number of data points, *k* is the lag. Since the ACF decays in an exponential fashion, and the PACF truncates abruptly after lag 2, we may tentatively identify the model for this time series as AR(2).

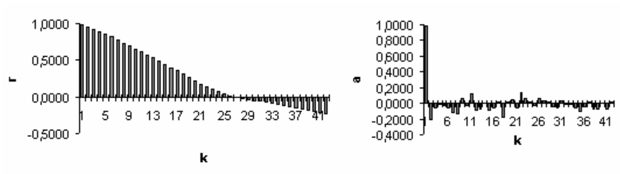

Figure 2: Autocorrelation function and partial autocorrelation function of the data for VAHOSTAV stock prices (period for analysis)

In the estimation step, we compute estimates for the parameters of the AR(2) model

$$y_t = \xi + a_1 y_{t-1} + a_2 y_{t-2} + \varepsilon_t \qquad (1)$$

$t = 1,2, \ldots, N-2$

or with of obvious matrix notation

$$\mathbf{y} = \mathbf{X}\mathbf{a} + \varepsilon$$

by OLS (Ordinary Least Squared)

$$\hat{\mathbf{a}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} \qquad (2)$$

In the diagnostic checking step, we test adequacy and closeness of fit of the model to the data by sample autocorrelation function of the residuals say

$$e_t = y_t - \hat{y}_t, t = 1,2, \ldots, \mathrm{N} - 2 \qquad (3)$$

The sample autocorrelation function of the residuals is shown in Fig. 3.

The modified Box-Pierce statistic Q is used for collectively testing the magnitudes of the residual autocorrelations for insignificance. The statistic is [5]

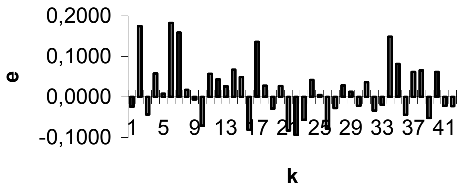

Figure 3: Sample autocorrelation function of the residuals for model (2)

$$Q = (N-d)\sum_{k=1}^{K} r_{ek}^2 \tag{4}$$

where r_{ek}^2 is the square of the residual autocorrelation coefficients, for lags k = 1, 2, ..., K, d is d-th differences of the data. For our stock price time series the Box-Pierce statistic for lag k = 42 was computed to be 27.78. This value is less then the critical chi square value of 55.7585 (degrees of freedom is 42 - 2=40, α = 0.005). Hence, we can conclude that the error terms are random and the model (1) is an adequate model. The parameters of AR(2) and AR(7) model were estimated by OLS an GLS methods and are shown in Tab 1.

Table. 1 OLS, GLS AND LSL ESTIMATES OF AR MODELS

Model	Order	Est.proc	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	$\hat{a}_4$	$\hat{a}_5$	$\hat{a}_6$	$\hat{a}_7$	RMSE*
(1)	2	OLS	1.113	-0.127				$\xi = 26.639$		67.758
(2)	7	GLS	-0.7513	-0.1701	-0.0230	-0.0128	-0.0028	-0.0472	0.0084	68.540

*ex post forecast period

3 Conventional and fuzzy neural network approach

The structure of a neural network is defined by its architecture, its activation functions and learning algorithm. While many variations are possible we suggested two models. The first one is an alternative of the most common form of the conventional neural network (CNN) which was suggested and discussed in [9]. This alternative of CNN is pictured in Fig. 4.

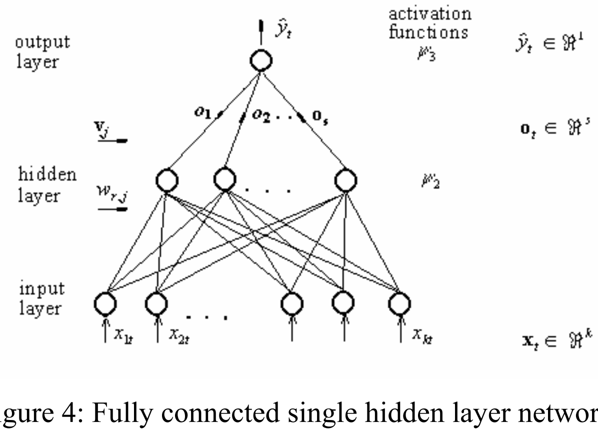

Figure 4: Fully connected single hidden layer network

The second one is the concept of fuzzy neural network (FNN) which was suggested and discussed in [8]. This alternative of FNN is pictured in Fig. 5.

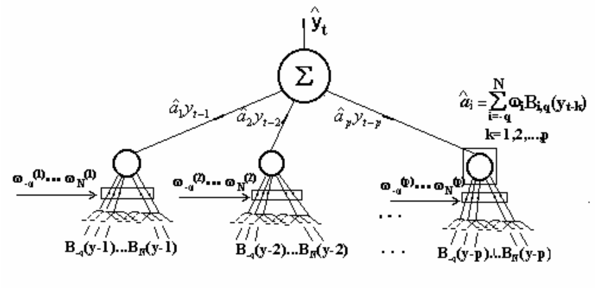

Figure 5: The neuro fuzzy system architecture

Fig. 4 shows a fully connected and strictly hierarchical ANN with variable number of inputs, further variable number of hidden layer units and one output unit. Processing units of the hidden layer have an activation function *S* - shaped *tanh*. Hidden layer weights w_{rj} are estimated from data according to the Back-Propagation learning technique.

In the FNN (Fig. 5) the inputs to the fuzzy neuron in hidden layer are fuzzy numbers denoted $B_{j,k,t}$, $j = 1, 2, ..., p$, k identifies the order of the B-spline basis functions. They express the neural input signals in terms of their membership functions based on B-spline basis functions of the data. This concept is often called as B-spline FNN [11]. The neuron in the output layer provides simply the computation of $y_t = \sum_{k=1}^{p} a_k y_{t-k}$ and produces output signal $\hat{y}_t$. The FNN was trained on the training data set by Back-Propagation algorithm. The RMSE's of our predictor models are shown in tab. 1.

Table 2: THE RMSE'S OF OUR PREDICTOR MODELS

Model	RMSE*
AR(2) - OLS estimates	67.7
AR(7) - GLS estimates	68,5
Conventional neural network	67.2
Fuzzy neural network	63.5

* Validation set

According to the results of our experiments, the predictor based on the CNN forecasting model is only slight better than the AR(2) model. As is stressed in [9], neural networks can outperform standard forecasting procedures at least for certain types of situations. Namely in situations where the relationship between inputs and outputs is highly nonlinear. The initial results of the FNN forecasting model are clearly better.

Acknowledgement

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