RECURSIVE IDENTIFICATION ALGORITHMS LIBRARY

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Abstract

This paper presents simple SIMULINK library for recursive parameter estimation of linear dynamic models ARX, ARMAX and OE. Several recursive identification methods were implemented in this library: Least Square Method (RLS), Recursive Leaky Incremental Estimation (RLIE), Damped Least Squares (DLS), Adaptive Control with Selective Memory (ACSM), Instrumental Variable Method (RIV), Extended Least Square Method (RELS), Prediction Error Method (RPEM) and Extended Instrumental Variable Method (ERIV). To cope with tracking the time-variant parameters several forgetting factor and modification of basic algorithm are taken into consideration.

1 Introduction

There exist many complex packages for system identification purposes in MATLAB and SIMULINK environment. These toolboxes provide solution to wide range of the problems from the area of system identification, e.g. System Identification Toolbox [11] and Continuous Identification Toolbox [6].

There also exist many special-purpose programs and libraries for MATLAB and SIMULINK, e.g. Idtool [3]. These simple tools provide solution to specific problems from the concrete part of the area of system identification.

The proposed Recursive Identification Algorithms Library (RIA) fall into category of simple libraries for SIMULINK environment and is designed for recursive estimation of the parameters of the linear dynamic models ARX, ARMAX and OE. The Recursive Identification Algorithms Library consists of several user-defined blocks. These blocks implement several recursive identification algorithms: Least Square Method (RLS) and its modifications, Recursive Leaky Incremental Estimation (RLIE), Damped Least Squares (DLS), Adaptive Control with Selective Memory (ACSM), Instrumental Variable Method (RIV), Extended Least Square Method (RELS), Prediction Error Method (RPEM) and Extended Instrumental Variable Method (ERIV). The Recursive Identification Algorithms Library can be used for simulation or real-time experiment (e.g. Real Time Toolbox) in educational process when it is possible to demonstrate the properties and behaviour of the recursive identification algorithms and forgetting factors under various conditions and can be also used in the identification part of self-tuning controllers.

2 Model structure

The basic step in identification procedure is the choice of suitable type of the model. General linear model takes the following form:

$$y(k)=\frac{B(q^{-1})}{A(q^{-1})F(q^{-1})}u(k)+\frac{C(q^{-1})}{A(q^{-1})D(q^{-1})}n(k) \tag{1}$$

where

$$
\begin{aligned}
A\left(q^{-1}\right) &= 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na} \\
B\left(q^{-1}\right) &= b_1 q^{-1} + b_2 q^{-2} + \ldots + b_{nb} q^{-nb} \\
C\left(q^{-1}\right) &= 1 + c_1 q^{-1} + \ldots + c_{nc} q^{-nc} \\
D\left(q^{-1}\right) &= 1 + d_1 q^{-1} + \ldots d_{nd} q^{-nd} \\
F\left(q^{-1}\right) &= 1 + f_1 q^{-1} + \ldots + f_{nf} q^{-nf}
\end{aligned}
\tag{2}
$$

are shift operators polynomials and $y(k)$, $u(k)$ are output and input signals. White noise $n(k)$ is assumed to have zero mean value and constant variance.

All linear models can be derived from general linear model by simplification. In the Recursive Identification Library following linear dynamic models are taken into consideration. These are ARX, ARMAX, OE models.

ARX model (*C*=*D*=*F*=1):¨

$$y(k) = \frac{B\left(q^{-1}\right)}{A\left(q^{-1}\right)} u(k) + \frac{1}{A\left(q^{-1}\right)} n(k) \tag{3}$$

ARMAX model (*D*=*F*=1):

$$y(k) = \frac{B\left(q^{-1}\right)}{A\left(q^{-1}\right)} u(k) + \frac{C\left(q^{-1}\right)}{A\left(q^{-1}\right)} n(k) \tag{4}$$

OE model (*A*=*C*=*D*=1):

$$y(k) = \frac{B\left(q^{-1}\right)}{F\left(q^{-1}\right)} u(k) + n(k) \tag{5}$$

3 Recursive parameter estimation

The recursive parameter estimation algorithms are based on the data analysis of the input and output signals from the process to be identified. Many recursive identification algorithms were proposed [10][16][17]. In this part several well-known recursive algorithms with forgetting factors implemented in Recursive Identification Algorithms Library are summarized.

3.1 RLS

This method can be used for parameter estimate of ARX model. The algorithm can be written in following form:

$$
\begin{aligned}
\hat{e}(k) &= \boldsymbol{y}(k) - \boldsymbol{\phi}^T(k)\hat{\boldsymbol{\Theta}}(k-1) \\
\boldsymbol{L}(k) &= \frac{\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)}{1 + \boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)} \\
\hat{\boldsymbol{\Theta}}(k) &= \hat{\boldsymbol{\Theta}}(k-1) + \boldsymbol{L}(k)\hat{e}(k) \\
\boldsymbol{C}(k) &= \boldsymbol{C}(k-1) - \boldsymbol{L}(k)\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)
\end{aligned}
\tag{6}
$$

where: $\boldsymbol{L}(k)$ denote gain matrix, $\boldsymbol{C}(k)$ is the covariance matrix of the estimated parameters, $\hat{\boldsymbol{\Theta}}(k)$ is the vector that contains the estimated parameters and $\boldsymbol{\phi}(k)$ is the data or regression vector

$$\hat{\boldsymbol{\Theta}}(k) = \left[a_1, \ldots a_{na}, b_1, \ldots, b_{nb}\right]^T \tag{7}$$

$$\boldsymbol{\phi}^T(k) = \left[y(k-1), \ldots, y(k-na), u(k-1), \ldots, u(k-nb)\right] \tag{8}$$

This RLS algorithm assumes that the parameters of the model process are constant. In many cases, however, the estimator will be required to track changes in a set of parameters. To cope with tracking the time-variant parameters some adjustment mechanism must be introduced in the previous basic equations. Several implementations have been proposed [10][16][9][4].

RLS with exponential forgetting

Covariance matrix is given by

$$\boldsymbol{C}(k)=\frac{1}{\lambda}\left(\boldsymbol{C}(k-1)-\frac{\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)}{\lambda+\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)}\right) \tag{9}$$

where $0<\lambda<1$ is forgetting factor.

The algorithm is convenient for identification and adaptive control of slowly varying systems. This method has the main disadvantages that when the inputs is not persistent, and as the old data is discarded in the estimation procedure, the matrix $\boldsymbol{C}(k)$ increases exponentially with rate λ. This is called estimator wind-up.

RLS with variable exponential forgetting

The variable exponential forgetting is given by relation

$$\lambda(k)=\lambda_0\lambda(k-1)+1-\lambda_0 \tag{10}$$

with $\lambda(0)=\lambda_0\in\langle 0{,}95;\,0{,}99\rangle$

This algorithm is convenient for identification of time-invariant systems and self-tuning controllers.

RLS with fixed directional forgetting

To solve the problem of estimator wind-up, an estimator with directional forgetting can be used. This estimator forgets the information only in the directions in which new information is gathered and assures the convergence of the estimations and avoids large changes in the parameters.

Covariance matrix is

$$\boldsymbol{C}(k)=\boldsymbol{C}(k-1)-\frac{\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)}{\varepsilon^{-1}+\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)} \tag{11}$$

and directional forgetting factor

$$\varepsilon(k-1)=\lambda'-\frac{1-\lambda'}{\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)} \tag{12}$$

where λ' can be chosen as in exponential forgetting algorithm.

RLS with adaptive directional forgetting

Detailed description of this algorithm can be found in [8].

$$\varepsilon(k)=\varphi(k)-\frac{1-\varphi(k)}{\xi(k-1)} \tag{13}$$

where

$$\xi(k-1)=\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\phi}(k) \tag{14}$$

The value of adaptive directional forgetting factor is

$$\varphi(k)=\left\{1+(1+\rho)[\ln(1+\xi(k-1))]+\left[\frac{(\upsilon(k-1)+1)\eta(k-1)}{1+\xi(k-1)+\eta(k-1)}-1\right]\frac{\xi(k-1)}{1+\xi(k-1)}\right\}^{-1} \tag{15}$$

and

$$
\begin{aligned}
\eta(k) &= \frac{\hat{e}^2(k)}{\lambda(k)} \\
\upsilon(k) &= \varphi(k)[\upsilon(k-1)+1] \\
\lambda(k) &= \varphi(k)\left[\lambda(k-1)+\frac{\hat{e}^2(k-1)}{1+\xi(k-1)}\right]
\end{aligned}
\tag{16}
$$

RLS with exponential forgetting matrix

This technique is able to cope with the cases where parameters have distinct rates of change in time. Here, is described a recursive estimation algorithm with exponential forgetting matrix factors in order to provide distinct information discounts for each parameter. The RLS with exponential forgetting matrix is governed by the following equations [10]:

$$\Lambda(k-1) = \Omega \boldsymbol{C}(k-1)\Omega^T \tag{17}$$

$$\boldsymbol{L}(k) = \frac{\Lambda(k-1)\boldsymbol{\phi}(k)}{1+\boldsymbol{\phi}^T(k)\Lambda(k-1)\boldsymbol{\phi}(k)} \tag{18}$$

$$\boldsymbol{C}(k) = \Lambda(k-1)\left(\boldsymbol{I}-\frac{\boldsymbol{\phi}(k)\boldsymbol{\phi}^T(k)\Lambda(k-1)}{1+\boldsymbol{\phi}^T(k)\Lambda(k-1)\boldsymbol{\phi}(k)}\right) \tag{19}$$

with

$$\Omega = \begin{bmatrix} \frac{1}{\sqrt{\lambda_1}} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{1}{\sqrt{\lambda_1}} \end{bmatrix} \tag{20}$$

representing a matrix with diagonal elements equal to square roots of the forgetting factors associated to each column of the regression vector $\boldsymbol{\phi}$.

RLS with constant trace algorithm

Constant trace algorithm could also be used to keep the matrix $\boldsymbol{C}(k)$ limited by scaling the matrix at each iterations in a way that trace of $\boldsymbol{C}(k)$ is constant. The regularized constant-trace algorithm is given by the following equations:

$$\hat{\boldsymbol{\Theta}}(k) = \hat{\boldsymbol{\Theta}}(k-1)+\boldsymbol{L}(k)\left(y(k)-\boldsymbol{\phi}^T(k)\hat{\boldsymbol{\Theta}}(k-1)\right) \tag{21}$$

$$\boldsymbol{L}(k) = \frac{\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)}{\lambda+\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)} \tag{22}$$

$$\bar{\boldsymbol{C}}(k) = \frac{1}{\lambda}\left(\boldsymbol{C}(k-1)-\frac{\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)}{1+\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)}\right) \tag{23}$$

$$\boldsymbol{C}(k) = c_1\frac{\bar{\boldsymbol{C}}(k)}{tr\left(\bar{\boldsymbol{C}}(k)\right)}+c_2\boldsymbol{I} \tag{24}$$

in which c_1 and c_2 have positive values given by,

$$\frac{c_1}{c_2} = 10000,\ \boldsymbol{\phi}^T(k)\boldsymbol{\phi}(k)c_1 \le 1 \tag{25}$$

Exponential Forgetting and Resetting Algorithm

This modification of RLS places upper and lower bounds on the trace of the covariance matrix while maintaining a robustly valued forgetting factor. The algorithm takes the following form:

$$\hat{\boldsymbol{\Theta}}(k)=\hat{\boldsymbol{\Theta}}(k-1)+\alpha\boldsymbol{L}(k)\hat{e}(k) \tag{26}$$

$$\boldsymbol{L}(k)=\frac{\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)}{\lambda+\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)} \tag{27}$$

$$\boldsymbol{C}(k)=\frac{1}{\lambda}\left[\boldsymbol{C}(k-1)-\boldsymbol{L}(k)\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\right]+\beta\boldsymbol{I}-\gamma\boldsymbol{C}(k-1)^2 \tag{28}$$

$$\sigma_{\min}\boldsymbol{I}\leq C(k-1)\leq\sigma_{\max}\boldsymbol{I} \qquad \forall k \tag{29}$$

$$\begin{gathered}\sigma_{\min}\approx\frac{\beta}{\alpha-\eta},\sigma_{\max}\approx\frac{\eta}{\gamma}+\frac{\beta}{\eta},\eta=\frac{1-\lambda}{\lambda}\\ \alpha=0,5;\beta=\gamma=0,005;\lambda=0,95;\\ \sigma_{\min}=0,01;\sigma_{\max}=10\end{gathered} \tag{30}$$

3.2 RWLS

The recursive weighted least square [15] where the weighting data $\boldsymbol{\phi}(k)$ is denoted as $q(k)$ becomes

$$\begin{gathered}\hat{e}(k)=\boldsymbol{y}(k)-\boldsymbol{\phi}^T(k)\hat{\boldsymbol{\Theta}}(k-1)\\ \boldsymbol{L}(k)=\frac{\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)}{\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)+\dfrac{\lambda}{q(k)}}\\ \hat{\boldsymbol{\Theta}}(k)=\hat{\boldsymbol{\Theta}}(k-1)+\boldsymbol{L}(k)\hat{e}(k)\\ \boldsymbol{C}(k)=\boldsymbol{C}(k-1)-\boldsymbol{L}(k)\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\end{gathered} \tag{31}$$

where $0<\lambda<1$ is forgetting factor.

3.3 RLIE

The recursive leaky incremental estimation [18] can be describes as follows:

$$\hat{\boldsymbol{\Theta}}(k)=\gamma\hat{\boldsymbol{\Theta}}(k-1)+\Gamma\hat{\boldsymbol{\Theta}}(k) \tag{32}$$

$$\Gamma\hat{\boldsymbol{\Theta}}(k)=\Gamma\hat{\boldsymbol{\Theta}}(k-1)+\boldsymbol{C}(k)\boldsymbol{\phi}(k)\left(y(k)-\gamma\boldsymbol{\phi}^T(k)\hat{\boldsymbol{\Theta}}(k-1)-\boldsymbol{\phi}^T(k)\Gamma\hat{\boldsymbol{\Theta}}(k-1)\right) \tag{33}$$

$$\boldsymbol{C}(k)=\frac{1}{\lambda}\left(\boldsymbol{C}(k-1)-\frac{\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)}{\lambda+\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)}\right) \tag{34}$$

where Γ denotes the stabilizing operator, defined as

$$\Gamma=1-\gamma q^{-1} \tag{35}$$

and $\gamma\in[0,1]$ is the stabilizing parameters which is preselected by the user.

3.4 DSL

Damped least squares (DLS) algorithm is an extended version of the recursive simple least square (RLS) algorithm.

The DLS criterion is

$$J(\hat{\boldsymbol{\Theta}})=\sum_{k=t-N}^{t}\lambda^{t-k}\left[y(k)-\boldsymbol{\phi}^T(k)\hat{\boldsymbol{\Theta}}(k)\right]^2+\left[\boldsymbol{\Lambda}_d(k)\left(\hat{\boldsymbol{\Theta}}(k)-\hat{\boldsymbol{\Theta}}(k-1)\right)\right]^2 \tag{36}$$

The weighting matrix $\boldsymbol{\Lambda}_d(k)$ is diagonal and weights the parameters variations. For an n-parameters model,

$$\Lambda_d(k-1)=diag[\alpha_1(k)\ \alpha_2(k)\ldots\alpha_n(k)] \tag{37}$$

A standard form of the DLS algorithm is given [12]

$$\hat{\boldsymbol{\Theta}}(k)=\hat{\boldsymbol{\Theta}}(k-1)+\boldsymbol{L}\left[y(k)-\hat{\boldsymbol{\Theta}}^T(k)\boldsymbol{\phi}(k)\right]+\boldsymbol{C}(k)\lambda(k)\boldsymbol{\Lambda}_d(k)\left[\hat{\boldsymbol{\Theta}}(k-1)-\hat{\boldsymbol{\Theta}}(k-2)\right] \tag{38}$$

$$\boldsymbol{L}(k)=\frac{\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)}{\lambda(k)+\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)} \tag{39}$$

$$\boldsymbol{C}(k)=\frac{1}{\lambda(k)}\left(\boldsymbol{C}'(k-1)-\frac{\boldsymbol{C}'(k-1)\boldsymbol{\phi}(k)\boldsymbol{\phi}^T(k)\boldsymbol{C}'(k-1)}{\lambda+\boldsymbol{\phi}^T(k)\boldsymbol{C}'(k-1)\boldsymbol{\phi}(k)}\right) \tag{40}$$

$$\boldsymbol{C}'(k)=\boldsymbol{C}'(k-1)-\sum_{i=1}^{n}\frac{\boldsymbol{C}'_{i-1}(k-1)r_i r_i^T\boldsymbol{C}'_{i-1}(k-1)\alpha'_i(k)}{1+r_i^T\boldsymbol{C}'_{i-1}(k-1)r_i\alpha'_i(k)} \tag{41}$$

$$\boldsymbol{C}'_1(k-1)=\boldsymbol{C}'(k-1)-\frac{\boldsymbol{C}'_{i-1}(k-1)r_i r_i^T\boldsymbol{C}'_{i-1}(k-1)\alpha'_i(k)}{1+r_i^T\boldsymbol{C}'_{i-1}(k-1)r_i\alpha'_i(k)} \tag{42}$$

$$\boldsymbol{C}'_0(k-1)=\boldsymbol{C}(k-1) \tag{43}$$

where $\boldsymbol{r}_{\mathrm{i}}$ are the succesive basic vectors, e.g.

$$r_i=[1..0\cdots0]^T \tag{44}$$

and

$$\alpha'_{\iota}(k)=\frac{\alpha_i(k)-\lambda(k)\alpha_i(k-1)}{\lambda(k)} \tag{45}$$

3.5 ACSM

Adaptive control with selective memory [7] updates parameter estimates only when there is new information present. The information increases and estimator eventually stops. The algorithm consists of several steps:

Step 0: Choose $r_0>0,\boldsymbol{\Theta}(0),\boldsymbol{C}(0)>0,1<M_0<\infty$

Set $r(0)=r_0>0,\sigma=1-\frac{1}{M_0},\varepsilon_0=\frac{1}{M_0}$.

Step 1:

$$r(k)=\max\left\{\sigma r(k-1)+(1-\sigma)\hat{e}(k-1)^2,r_0\right\} \tag{46}$$

where $\hat{e}(k)=\boldsymbol{y}(k)-\boldsymbol{\phi}^T(k)\hat{\boldsymbol{\Theta}}(k-1)$

Step 2: Set $B(k)=0$ and

$$A(k)=\begin{cases}1 & if\ \dfrac{\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)}{r(k)}\geq\varepsilon_0\\ 0 & otherwise\end{cases} \tag{47}$$

Step 3: If $A(k)=0$, set

$$B(k)=\begin{cases}1 & if\ r(k)\geq\max_{1\leq i\leq k}r(k-i)\\ 0 & otherwise\end{cases} \tag{48}$$

Set $\Delta(k)=A(k)+B(k)$

Step 4:

$$\hat{\boldsymbol{\Theta}}(k) = \hat{\boldsymbol{\Theta}}(k-1) + \Delta(k)\frac{\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)}{r(k) + \boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)}\hat{e}(k) \tag{49}$$

Step 5:

$$\boldsymbol{C}(k) = \boldsymbol{C}(k-1) - \Delta(k)\frac{\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)}{r(k) + \boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{\phi}(k)} \tag{50}$$

Step 6: Set $k = k + 1$ and go to step 1

3.6 RIV

It can be shown that if the process does not meet the noise assumption made by the ARX model, the parameters are estimated biased and non-consistent. This problem can be avoided using instrumental variable method.

The algorithm takes the form

$$\begin{aligned}
\hat{e}(k) &= \boldsymbol{y}(k) - \boldsymbol{\phi}^T(k)\hat{\boldsymbol{\Theta}}(k-1) \\
\boldsymbol{L}(k) &= \frac{\boldsymbol{C}(k-1)\boldsymbol{z}(k)}{1 + \boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)\boldsymbol{z}(k)} \\
\hat{\boldsymbol{\Theta}}(k) &= \hat{\boldsymbol{\Theta}}(k-1) + \boldsymbol{L}(k)\hat{e}(k) \\
\boldsymbol{C}(k) &= \boldsymbol{C}(k-1) - \boldsymbol{L}(k)\boldsymbol{\phi}^T(k)\boldsymbol{C}(k-1)
\end{aligned} \tag{51}$$

where: $\boldsymbol{L}(k)$ denote gain matrix, $\boldsymbol{C}(k)$ is the covariance matrix of the estimated parameters, $\hat{\boldsymbol{\Theta}}(k)$ is the vector that contains the estimated parameters, $\boldsymbol{\phi}(k)$ is the data or regression vector, $\boldsymbol{z}(k)$ is instrumental variable

$$\hat{\boldsymbol{\Theta}}(k) = [a_1, \ldots a_{na}, b_1, \ldots, b_{nb}]^T \tag{52}$$

$$\boldsymbol{\phi}^T(k) = [y(k-1), \ldots, y(k-na), u(k-1), \ldots, u(k-nb)] \tag{53}$$

Choice of instrumental variable determines behaviour of the IV method in usage. Some common choices for generating instruments are proposed in [16].

Typical choice of model independent instrumental variable is

$$\boldsymbol{z}(k) = [u(k-1), \ldots, u(k-na-nb)]^T \tag{54}$$

and model dependent instrument is

$$z(k) = [y_u(k-1), \ldots, y_u(k-na), u(k-1), \ldots, u(k-nb)]^T \tag{55}$$

where $y_u(k-1)$ is generated by calculating following difference equation with current parameter estimates

$$y_u(k) = \hat{b}_1(k)u(k-1) + \ldots + \hat{b}_{nb}(k)u(k-nb) - \hat{a}_1(k)y_u(k-1) + \ldots + \hat{a}_{na}(k)y_u(k-na) \tag{56}$$

3.7 RELS

This method is used for parameter estimations of ARMAX model. Formally it takes the same form as RLS. However, the regression and parameter vector are different.

Parameter vector

$$\hat{\boldsymbol{\Theta}}(k) = [a_1, \ldots a_{na}, b_1, \ldots, b_{nb}, c_1, \ldots, c_{nc}]^T \tag{57}$$

Regression vector

$$\boldsymbol{\phi}^T(k) = [y(k-1), \ldots, y(k-na), u(k-1), \ldots, u(k-nb), \eta(k-1), \ldots, \eta(k-nc)] \tag{58}$$

or

$$\boldsymbol{\phi}^T(k) = [y(k-1), \ldots, y(k-na), u(k-1), \ldots, u(k-nb), \hat{e}(k-1), \ldots, \hat{e}(k-na)] \tag{59}$$

where $\eta(k)$ denotes the residual and $\hat{e}(k)$ is the prediction error.

It usually speeds up the convergence of the RELS algorithm if the residuals (*a posteriori*) rather than the prediction errors (*a priori*) are used.

3.8 ERIV

This method ensures improved accuracy and greater speed of convergence than RIV. The method is based on choice of instruments vector which has more elements than there are parameters in the model to be estimated. Derivation of this algorithm can be found in [16]. Instruments can be chosen according to [2][16].

The set of equations describe this algorithm

$$\begin{aligned}
\hat{\boldsymbol{\Theta}}(k) &= \hat{\boldsymbol{\Theta}}(k-1) + \boldsymbol{L}(k)\left(\boldsymbol{v}(k) - \boldsymbol{\Phi}^T(k)\hat{\boldsymbol{\Theta}}(k-1)\right) \\
\boldsymbol{L}(k) &= \boldsymbol{P}(k-1)\boldsymbol{\Phi}(k)\left(\boldsymbol{\Lambda}(k) + \boldsymbol{\Phi}^T(k)\boldsymbol{P}(k-1)\boldsymbol{\Phi}(k)\right)^{-1} \\
\boldsymbol{\Phi}(k) &= \left[\boldsymbol{w}(k) \quad \boldsymbol{\phi}(k)\right] \\
\boldsymbol{w}(k) &= \boldsymbol{R}^T(k-1)\boldsymbol{z}(k) \\
\Lambda(k) &= \begin{bmatrix} -\boldsymbol{z}^T(k)\boldsymbol{z}(k) & 1 \\ 1 & 0 \end{bmatrix} \\
\boldsymbol{v}(k) &= \begin{bmatrix} \boldsymbol{z}^T(k)\boldsymbol{r}(k-1) \\ \boldsymbol{y}(k) \end{bmatrix} \\
\boldsymbol{R}(k) &= \boldsymbol{R}(k-1) + \boldsymbol{z}(k)\boldsymbol{\phi}^T(k) \\
\boldsymbol{r}(k) &= \boldsymbol{r}(k-1) + \boldsymbol{z}(k)\boldsymbol{y}(k) \\
\boldsymbol{P}(k) &= \boldsymbol{P}(k-1) - \boldsymbol{L}(k)\boldsymbol{\Phi}^T(k)\boldsymbol{P}(k-1)
\end{aligned} \tag{60}$$

3.9 RPEM

The recursive prediction error method (RPEM) allows the online identification of all linear model structure. Since all model structure except ARX are nonlinearly parameterized, no exact recursive algorithm can exist; rather some approximations must be made [13][14][16]. In fact, the RPEM can be seen as a nonlinear least squares Gauss-Newton method.

The Gauss-Newton technique is based on the approximation of the Hessian by the gradients. Thus, the RPEM requires the calculation of the gradient $\psi(k)$ of the model output with respect to its parameters:

$$\boldsymbol{\psi}^T(k) = \frac{\partial\,\hat{y}(k)}{\partial\,\boldsymbol{\Theta}(k)} = \begin{bmatrix} \dfrac{\partial\,\hat{y}(k)}{\partial\,\Theta_1(k)} & \dfrac{\partial\,\hat{y}(k)}{\partial\,\Theta_2(k)} & \cdots & \dfrac{\partial\,\hat{y}(k)}{\partial\,\Theta_n(k)} \end{bmatrix} \tag{61}$$

RPEM algorithm takes the form

$$\begin{aligned}
\hat{e}(k) &= y(k) - \boldsymbol{\phi}^T(k)\hat{\boldsymbol{\Theta}}(k-1) \\
\boldsymbol{L}(k) &= \frac{\boldsymbol{P}(k-1)\boldsymbol{\psi}(k)}{1 + \boldsymbol{\psi}^T(k)\boldsymbol{P}(k-1)\boldsymbol{\psi}(k)} \\
\hat{\boldsymbol{\Theta}}(k) &= \hat{\boldsymbol{\Theta}}(k-1) + \boldsymbol{L}(k)\hat{e}(k) \\
\boldsymbol{P}(k) &= \boldsymbol{P}(k-1) - \boldsymbol{L}(k)\boldsymbol{\psi}^T(k)\boldsymbol{P}(k-1)
\end{aligned} \tag{62}$$

where $\boldsymbol{P}(k)$ denotes covariance matrix.

The model structure will influence the way in which the quantities $\hat{e}(k)$ and $\psi(k)$ in the algorithm are computed from data and the previously computed parameter estimate.

4 RECURSIVE IDENTIFICATION ALGORITHMS LIBRARY (RIA)

The Recursive Identification Algorithm Library is designed for recursive parameter estimation of linear dynamics model ARX, ARMAX, OE using recursive identification methods: Least Square Method (RLS), Recursive Leaky Incremental Estimation (RLIE), Damped Least Squares (DLS), Adaptive Control with Selective Memory (ACSM), Instrumental Variable Method (RIV), Extended Least Square Method (RELS), Prediction Error Method (RPEM) and Extended Instrumental Variable Method (ERIV).

The Recursive Identification Algorithm Library is depicted in Fig. 1. The Library consists of 18 user-defined blocks and is designed for MATLAB&SIMULINK environment. Each block is realized as an s-function.

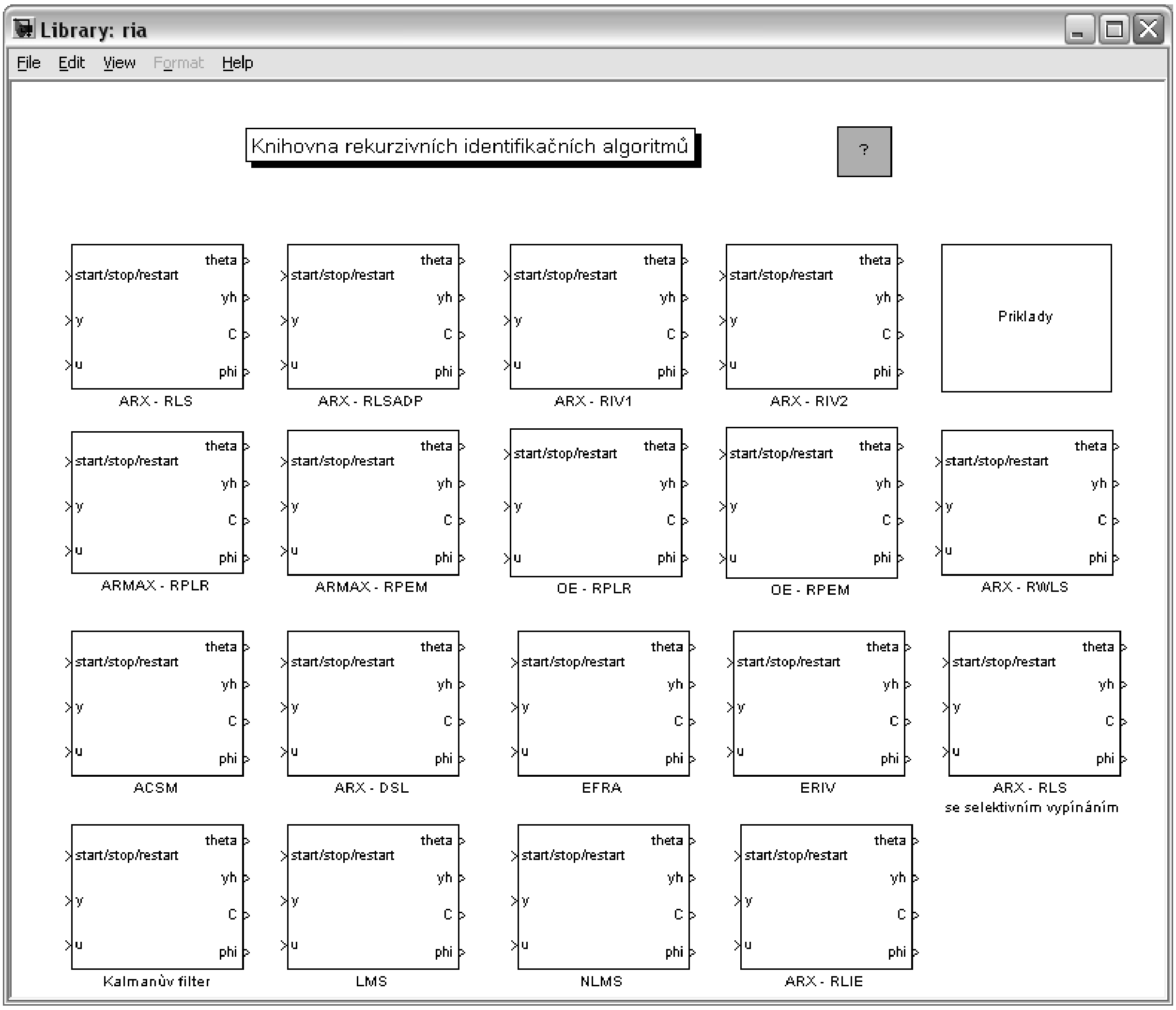
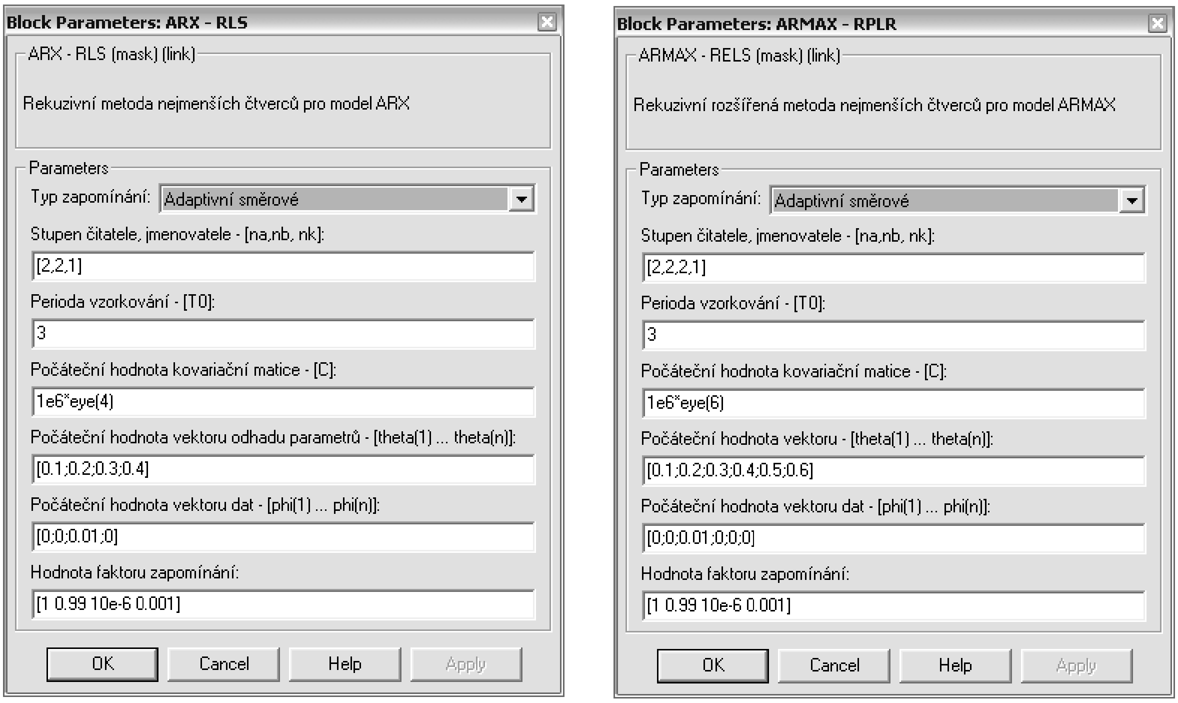


Figure 1: Recursive Identification Algorithms Library

Each block is masked by user-defined dialog. Several necessary input parameters should be input through this dialog. These are: type of forgetting factor and its value, degrees of polynomials, sampling period, initial values of parameter estimate, covariance matrix and data vector, etc. Each block also contains the help describes the meaning of each parameter, inputs and outputs and used recursive identification algorithms. Example of input dialog is shown in Fig. 2.



Block Parameters: ARX - RLS

ARX - RLS (mask) (link)

Rekuzivní metoda nejmenších čtverců pro model ARX

Parameters

Typ zapomínání: Adaptivní směrové

Stupen čitatele, jmenovatele - [na,nb, nk]: [2,2,1]

Perioda vzorkování - [T0]: 3

Počáteční hodnota kovariační matice - [C]: 1e6*eye(4)

Počáteční hodnota vektoru odhadu parametrů - [theta(1) ... theta(n)]: [0.1;0.2;0.3;0.4]

Počáteční hodnota vektoru dat - [phi(1) ... phi(n)]: [0;0;0.01;0]

Hodnota faktoru zapomínání: [1 0.99 10e-6 0.001]

Block Parameters: ARMAX - RPLR

ARMAX - RELS (mask) (link)

Rekuzivní rozšířená metoda nejmenších čtverců pro model ARMAX

Parameters

Typ zapomínání: Adaptivní směrové

Stupen čitatele, jmenovatele - [na,nb, nk]: [2,2,2,1]

Perioda vzorkování - [T0]: 3

Počáteční hodnota kovariační matice - [C]: 1e6*eye(6)

Počáteční hodnota vektoru - [theta(1) ... theta(n)]: [0.1;0.2;0.3;0.4;0.5;0.6]

Počáteční hodnota vektoru dat - [phi(1) ... phi(n)]: [0;0;0.01;0;0;0]

Hodnota faktoru zapomínání: [1 0.99 10e-6 0.001]

Figure 2: Input dialog of the identification block

Input/output data from object under identification process are inputs to the identification block. Another input (start/stop/restart) is used for control the identification algorithm. This input provides possibility of start, stop and restart the identification algorithm in selected instant of time. Outputs of the block are estimate of parameter vector, one-step prediction of output of model, covariance matrix and data vector.

Example of application of the identification block in the model is illustrated in Fig. 3.

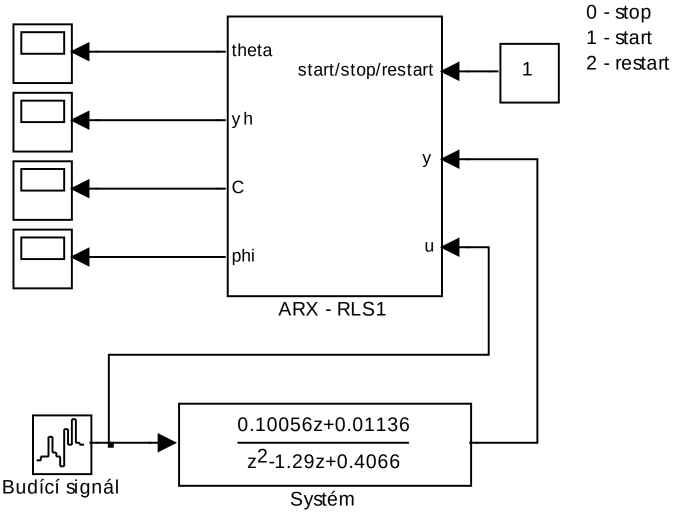


Figure 3: Example of application of identification block

5 Conclusion

The Recursive Identification Algorithm Library is designed for recursive parameter estimation of linear dynamics model ARX, ARMAX, OE using recursive identification methods. The library can be used e.g. in identification part of self-tuning controller or in educational process when it is possible to demonstrate the properties and behaviour of the recursive identification algorithms and forgetting factors under various conditions.

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