HYBRID PREDICTIVE CONTROLLER

J. Paulusová, M. Dúbravská

Institute of Control and Industrial Informatics,

Slovak University of Technology, Faculty of Electrical Engineering and Information Technology,

Ilkovičova 3, 812 19 Bratislava, Slovak Republic

Abstract

In this paper hybrid neuro-fuzzy model based predictive control (HNFMBPC) is addressed, proposed and tested. The proposed hybrid neuro-fuzzy convolution model consists of a steady-state neuro-fuzzy model and a gain independent impulse response model. The proposed model is tested in model based predictive control of the concentration control in the chemical reactor, manipulating its flow rate. The paper deals with theoretical and practical methodology, offering approach for intelligent fuzzy robust control design and its successful application.

1 Introduction and preliminaries

In the beginning, the predictive control algorithms were applied in the process technology. Today the predictive controllers are used in many areas where a high quality control is required. Predictive control is a control strategy that is based on the prediction of the plant output over the extended horizon in the future, which enables the controller to predict future changes of the measurement signal and to base control actions on the prediction. Advanced predictive control techniques use fuzzy logic and neural networks for modeling and control. The new intelligent control methods based on neuro-fuzzy approaches used in predictive control algorithms are an efficient tool for handling plant with complex dynamics as well as unstable inverse systems plant model miss-matches, different uncertainties, etc. Due to application of the strategy of using fuzzy systems with learning abilities of neural networks the algorithms allow to obtain a higher accuracy of the required output in a much shorter time compared to classical systems.

The future process output is predicted over the prediction horizon using hybrid neuro-fuzzy convolution model (HNFCM). The proposed HNFCM can be considered as a gain-scheduled convolution model. This paper shows the advantages of a combination of a static nonlinearity and gain-independent dynamic part. The steady-state behavior of the process is represented by a neuro-fuzzy model structure which is based on linguistic knowledge about the steady-state behavior of the process. The parameters of the rule consequents are identified using input and output data gathered from the process. The dynamic part is represented by an impulse response model. The proposed HNFCM is applied in model predictive control.

The paper is organized as follows: First, design of the hybrid neuro-fuzzy convolution model is briefly introduced in Section 2. Then, the neuro-fuzzy model based predictive controller is discussed in Section 3. The reliability and effectiveness of the presented method is shown on one application in Section 4 - control of the concentration control in the chemical reactor by manipulating its inlet flow rate. Summary and conclusions are given in Section 5.

2 The hybrid neuro-fuzzy convolution model (HNFCM)

The output of the model can be formulated as [1]

$$y_m(k+1) = y_s + K(u_s, x_2, \dots, x_n).\sum_{i=1}^{N} g_i(x_2, \dots, x_n).(u(k-i+1) - u_s) \tag{1}$$

where $y_s + K(u_s, x_2, \dots, x_n)$ is steady-state part, which is described by Takagi-Sugeno fuzzy model and $\sum_{i=1}^{N} g_i(x_2, \dots, x_n).(u(k-i+1) - u_s)$ is dynamic part of model (the impulse response model). The gain independent impulse response model is $g_i(x_2,...,x_n)$, the previous input values are $u(k\text{-}i\text{-}1)$ over N horizon, K is steady-state gain, u_s and y_s are steady-state input and output, $x_2, \dots, x_n$ are other operating parameters having effects on the steady-state output.

The convolution is multiplied by steady-state gain

$$K = \frac{\partial f(u_s, x_2, \ldots, x_n)}{\partial u_s} \quad (2)$$

2.1 The steady-state part of HNFCM

The steady state part is described by Takagi-Sugeno (T-S) fuzzy model. T-S models are suitable for modeling a large class of nonlinear systems [4]. A nonlinear discrete system can be expressed by T-S fuzzy model with n rules. The i-th rule of the T-S model is described as follows:

$$R^i : if\ x_1\ is\ A_{1,i}\ and \ldots and\ x_n\ is\ A_{n,i}\ then\ y_s = d_i \quad (3)$$

where n is the number of inputs, $\mathbf{x}=[x_1,...,x_n]^T$ is a vector of inputs of the model, $A_{j,i}(x_j)$ is the i=1,2,...,M_j –th antecedent fuzzy set referring to the j-th input, where M_j is the number of the fuzzy set on the j-th input domain. The first element of the input vector is the steady-state input $x_1=u_s$.

The output is computed as the weighted average of the individual rules' consequents

$$y_s = \frac{\sum_{i=1}^{m} \mu_i d_i}{\sum_{i=1}^{m} \mu_i} \quad (4)$$

where the weights $0 \le \mu_i \le 1$ are computed as $\mu_i = \prod_{j=1}^{m} A_{ji}(x_j)$, where $\prod$ is fuzzy operator, usually been applied as the *min* or the *product* operator and m is number of rules. Triangular membership functions were employed for each fuzzy linguistic value as shown in Fig. 1.

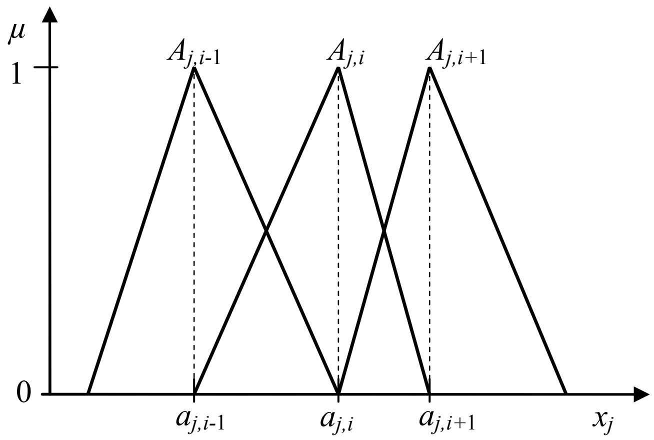

Figure 1: Membership function used for the fuzzy model

The membership functions are defined as follows:

$$A_{j,i}(x_j) = \frac{x_j - a_{j,i-1}}{a_{j,i} - a_{j,i-1}}, \qquad a_{j,i-1} \le x_j < a_{j,i}$$
$$A_{j,i}(x_j) = \frac{a_{j,i+1} - x_j}{a_{j,i+1} - a_{j,i}}, \qquad a_{j,i} \le x_j < a_{j,i+1} \quad (5)$$

where $x_j \in (a_{j,m_j}, a_{j,m_j+1}\rangle$.

The gain of the steady-state fuzzy model can be computed as

$$K_j = \frac{\partial y_s}{\partial u_s} = \sum_{i=m_j}^{m_j+1} \left[\left(\frac{\Gamma_{i-1}(u_s)}{a_{1,i} - a_{1,i-1}} - \frac{\Gamma_i(u_s)}{a_{1,i+1} - a_{1,i}} \right) \prod_{j=2}^{n} A_{j,i}(x_j) d_i \right], \qquad K = \sum_{j=1}^{n} K_j \quad (6)$$

where $\Gamma_i = 1 \quad if \quad u_s \in (a_{1,i}, a_{1,i+1}\rangle$

$\Gamma_i = 0 \quad if \quad u_s \notin (a_{1,i}, a_{1,i+1}\rangle$

2.2 The dynamic part of HNFCM

The dynamic state part is described by the impulse response model (IRM). The parameters of the discrete IRM g_i (i=0,…,N, where N is the model horizon) can be easily calculated from the input-output data (u_i and y_i) from the process.

$$y(k) = \sum_{i=0}^{k} g_i u(k-i) \tag{7}$$

In matrix form

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} u_0 & 0 & \cdots & 0 & \cdots \\ u_1 & u_0 & \cdots & 0 & \cdots \\ \vdots & \vdots & & & \vdots \\ u_N & u_{N-1} & \cdots & u_0 & \cdots \end{bmatrix} \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_N \end{bmatrix}$$

The parameters are given as follows

$$g^* = \left(U^T U\right)^{-1} U^T y \tag{8}$$

3 The hybrid neuro-fuzzy model based predictive controller (HNFMBPC)

The nonlinear HNFCM can be easily applied in model based predictive control scheme. In most cases, the difference between system outputs and reference trajectory is used with combination of a cost function on the control effort. A general objective function is the following quadratic form

$$J = \sum_{i=1}^{p} [\hat{y}(k+i\,|\,k) - r(k+i)]^2 \Gamma_y + \sum_{i=1}^{m} (\Delta u(k+i-1))^2 \Gamma_u \tag{9}$$

Here, r is desired set point, Γ_u ($\Gamma_u = \gamma . K^2$) and Γ_y are weight parameters, determine the relative importance of the different terms in the cost function, u and Δu are the control signal and its increment, respectively. Parameter p is the length of the prediction horizon and m is the length of the control horizon. Output predicted by the nonlinear fuzzy model is $\hat{y}(k)$.

$$\hat{y}(k) = K \sum_{i=1}^{\infty} s_i \Delta u(k-i) \tag{10}$$

where $s_i = \sum_{j=1}^{i} g_j$ are the step response coefficients and the change on the control variable is $\Delta u(k) = u(k) - u(k-1)$.

The model predictions along the prediction horizon p are

$$\hat{y}(k+j\,|\,k) = K \sum_{i=1}^{\infty} s_i \Delta u(k+j-i) + e(k+j\,|\,k) \tag{11}$$

Disturbances are considered to be constant between sample instants

$$e(k+j\,|\,k) = y(k\,|\,k) - K \sum_{i=1}^{\infty} s_i \Delta u(k+j-i) \tag{12}$$

where $y(k|k)$ represents the measured value of the process output at time k.

The model output is

$$\hat{y}(k+j\,|\,k) = K \sum_{i=1}^{N} s_i \Delta u(k+j-i) + f(k+j\,|\,k) \tag{13}$$

where

$$f(k+j\,|\,k) = y(k\,|\,k) + K \sum_{i=1}^{N} (s_{k+1} - s_i) \Delta u(k-i) \tag{14}$$

The prediction of the process output along the length of the prediction horizon, can be written compactly using matrix notation

$$\hat{y}(k) = KS\Delta u(k) + f(k) \tag{15}$$

where

$$S = \begin{bmatrix} s_1 & 0 & \cdots & 0 \\ s_2 & s_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ s_m & s_{m-1} & \cdots & s_1 \\ \vdots & \vdots & \ddots & \vdots \\ s_p & s_{p-1} & \cdots & s_{p-m+1} \end{bmatrix}_{p \times m} \tag{16}$$

Matrix S is called the system's dynamic matrix [5]. By minimizing its objective function (9) the optimal solution is then given

$$\Delta u(k) = \frac{1}{K}(S^T \Gamma_y S + \gamma I)^{-1} S^T \Gamma_y e(k) \tag{17}$$

In many control applications the desired performance cannot be expressed solely as a trajectory following problem. Many practical requirements are more naturally expressed as constraints on process variables such as manipulated variable constraints, manipulated variable rate constraints or output variable constraints. The solution calls into existence of quadratic programming solution of the control problem.

3.1 The algorithm for HNFCM based control

The algorithm has the following steps:

1. Calculation impulse response model g_i from (8),
2. Calculation of u_s from y_s=$y(k)$, considering the inversion of the fuzzy model,
3. Calculation of the value of the steady-state gain K by (6),
4. Calculation of S by (16) and e by (12),
5. Calculation of the controller output from the first element of the calculated Δu vector generated from (17).

4 Case study and simulation results

4.1 Case study

The application considered involves an isothermal reactor in which the Van Vusse reaction kinetic scheme is carried out. In the following analysis, A is the educt, B the desired product, C and D are unwanted byproducts.

$$\begin{aligned} &A \xrightarrow{k_1} B \xrightarrow{k_2} C \\ &2A \xrightarrow{k_3} D \end{aligned} \tag{18}$$

From a design perspective the objective is to make k_2 and k_3 small in comparison to k_1 by appropriate choice of catalyst and reaction conditions. The concentration of B in the product may be controlled by the manipulating the inlet flow rate and/or the reaction temperature.
The educt flow contains only cyclopentadiene in low concentration, C_{Af}. Assuming constant density and an ideal residence time distribution within the reactor, the mass balance equations for the relevant concentrations of cyclopentadiene and of the desired product cyclopentanol, C_A and C_B, are as follows.

$$\begin{aligned} \dot{C}_A &= -k_1 C_A - k_3 C_A^2 + \frac{F}{V}(C_{Af} - C_A) \\ \dot{C}_B &= k_1 C_A - k_2 C_B - \frac{F}{V} C_B \\ y &= C_B \end{aligned} \tag{19}$$

This example has been considered by a number of researchers as a benchmark problem for evaluating nonlinear process control algorithm. By normalizing the process variables around the following operating point and substituting the values for the physical constants, the process model becomes:

$$\begin{aligned} \dot{x}_1(k) &= -50x_1(k) - 10x_1^2(k) + u(10 - x_1(k)) \\ \dot{x}_2(k) &= 50x_1(k) - 100x_2(k) + u(-x_2(k)) \\ y(k) &= x_2(k) \end{aligned} \tag{20}$$

where the deviation variable for the concentration of component A is denoted by x_1, the concentration of component B by x_2, and the inlet flow rate by u. The simulation scheme of this process is in Fig. 2.

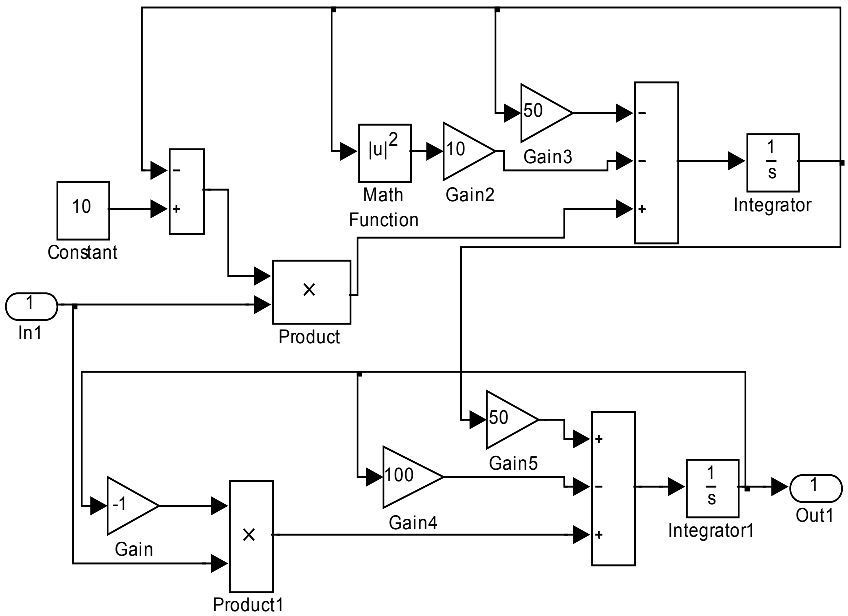

Figure 2: Simulation scheme for the nonlinear process described by (20)

4.2 Simulation results

Graphic menu was created in GUIDE, graphic toolbox of MATLAB for creation of user applications. Menu deals with tuning of parameters of predictive controllers: predictive horizon (p), control horizon (m) and gamma (γ). User menu is displayed on Fig. 3. In this application model with different membership functions can be chosen. Description of HNFCM with Gaussian, trapezoidal and bell-type membership functions can be found in [6].

The comparison of HNFCM with the nonlinear plant is shown in Fig. 3. Steady-state part of HNFCM was created in ANFIS editor in MATLAB. Time responses of the controlled and reference variables under HNFMBPC with the effect of tuning parameter γ are shown in Fig. 4 and Fig. 5.

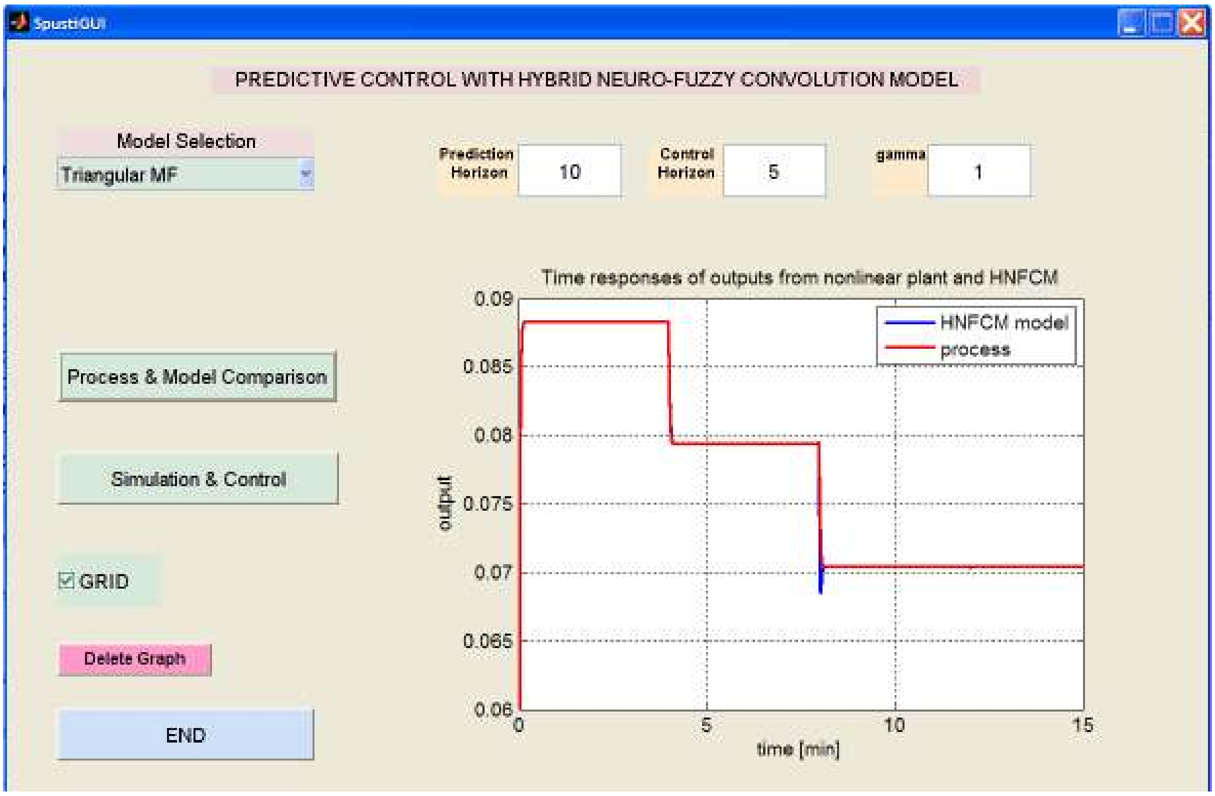

Figure 3: Time responses of output from the nonlinear plant and the HNFCM model

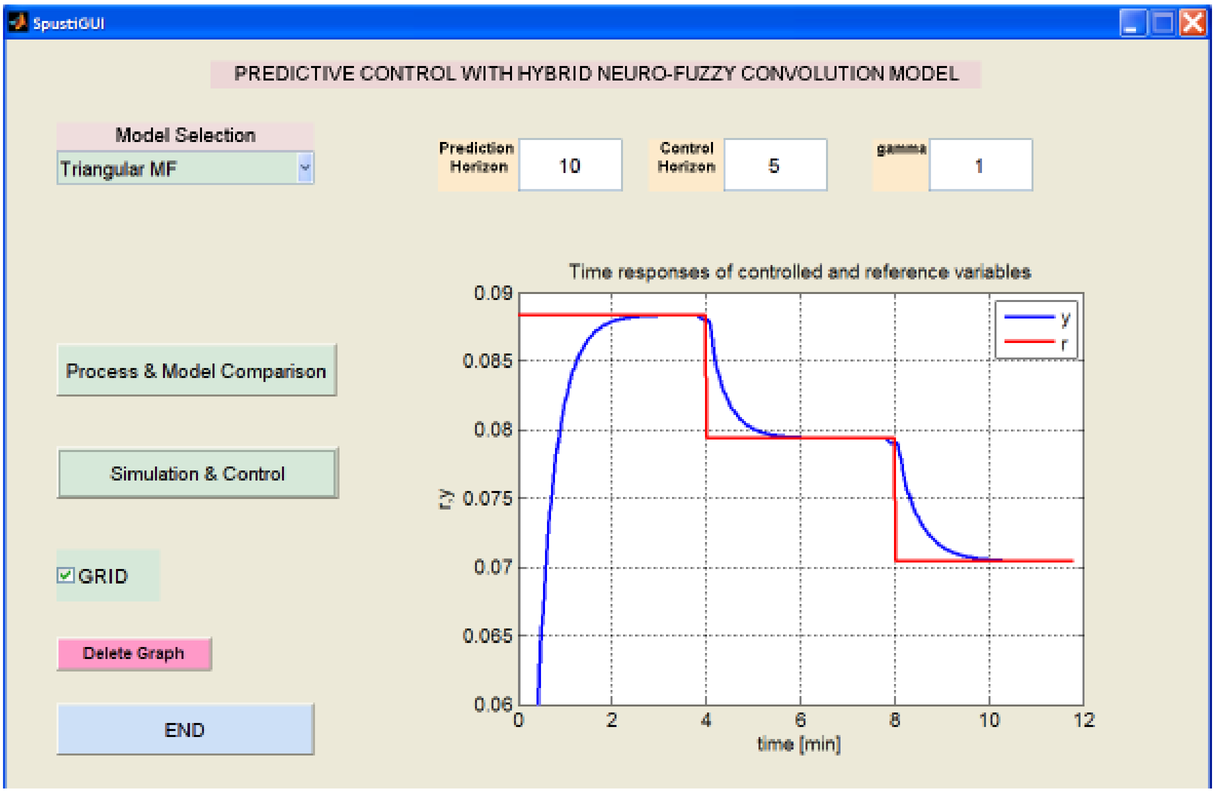

Figure 4: Time responses of the controlled and reference variables under HNFMBPC (m=5, p=10, Γ_y=K, γ=1)

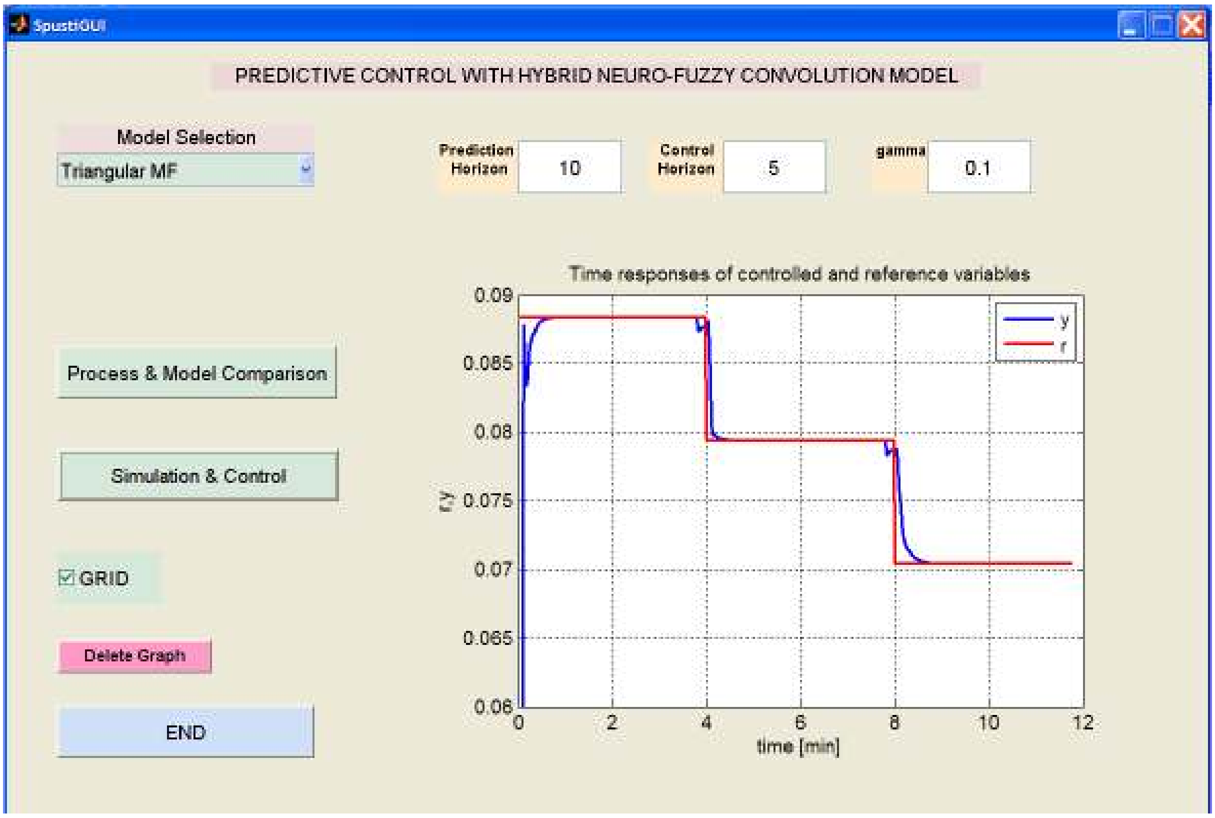

Figure 5: Time responses of the controlled and reference variables under HNFMBPC (m=5, p=10, Γ_y=K, γ=0.1)

5 Conclusion

The HNFMBPC uses the advantage of fuzzy systems in the representation of the steady-state behavior of the system. Other advantage is that it tries to combine knowledge about the system in form of a priori knowledge and measured data in the identification of a control relevant model.

Simulation example illustrates the potential offered by the HNFCMBPC.

This paper has been supported by the Slovak Scientific Grant Agency, Grant No. 1/0544/09.

References

[1] Abonyi, J., Bódizs, Á., Nagy, L., Szeifert, F.: Hybrid fuzzy convolution model and its application in predictive control. *Chemical Engineering Research and Design* 78 (2000), pp. 597-604.
[2] Paulusová, J., Kozák, Š.: Robust Predictive Fuzzy Control. *7th Portuguese Conference on Automatic Control, CONTROLO'2006*, Lisbon, Portugal: 11-13 September 2006, MA-7-1
[3] Paulusová, J., Kozák, Š.: The comparison of the conventional controllers with fuzzy controllers. Tatranské Matliare, Slovac Republic, May 31-June 3, 1999, 168-171
[4] Paulusová, J., Kozák, Š.: Nonlinear model-based predictive control. *Control Systems Design 2003 CSD'03*. Bratislava, Slovak Republic: 7-10 September, 2003, pp. 171-175.
[5] Paulusová, J., Kozák, Š.: Robust and Fuzzy Dynamic Matrix Control Algorithm. *5th International Carpathian Control Conference 2004*, Zakopane, Poland: 25-28 May, pp. 145-152
[6] Paulusová, J., Dúbravská, M.: Predictive Control of Nonlinear Process. *International Slovak Society for Cybernetics and Informatics*, Vyšná Boca, Slovak Republic: 10-13 February 2010, 20_Paulusova_Dubravska.pdf

Jana Paulusová
tel.: +421 2 60291262
mail: jana.paulusova@stuba.sk

Mária Dúbravská
mail: maria.dubravska@stuba.sk