# THE INTRODUCTION OF COMPUTER LESSONS IN COURSE OF ELECTROMAGNETIC WAVES, ANTENNAS AND TRANSMISSION LINES

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**Abstract**

**A conventional concept in courses of electromagnetic field theory largely leans on the frequency-domain description of pertaining phenomena. Since the electromagnetic (EM) field does evolve in space-time, the latter description can lead to misleading notions about a physical EM field behavior. For this reason, the education of EM field specialists should be focused mainly on the space-time domain. In this paper, examples of basic EM phenomena as prepared in the COMSOL Multiphysics® for teaching in the Department of Radio Electronics are presented.**

## 1 Introduction

The COMSOL Multiphysics® is used as computing and visualization tool for the space-time analysis of the EM field of the fundamental subjects as plane-wave, cylindrical-wave and diffraction of waves on dielectric and conductors (see Fig. 2). The aim is mediation of the knowledge and skills in numerical modeling of EM fields in the time domain. Students will be acquainted with the finite element method and the space-time formulation of EM field problems. They will become familiar with the modeling procedure in the COMSOL Multiphysics® including the work with a mesher, dealing with solver parameters and handling the post-processing tools. Emphasis is placed on the illustrative character of the results and mathematical clarity of their formulation.

As an example of our approach, a solution of two-dimensional wave equation is discussed in connection with the conventional concept of description as given in the majority of EM-aimed textbooks (for example, [1], [2]) and with the application of Hadamard's method of steepest descent [4]. A number of illustrative numerical examples provided by COMSOL Multiphysics® are given.

## 2 Some notes on solutions of a scalar two-dimensional wave equation

This section is aimed at finding a scalar wave generated by a line source placed at $x = y = 0$. The corresponding wave function $u = u(x,y,t)$ satisfies the two-dimensional scalar wave equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2} = -\delta(x,y) f(t) \ , \tag{1}$$

where $\delta(x,y)$ is two-dimensional Dirac's distribution, $c_0$ is a wave front velocity and $f(t)$ denotes strength of the line source. It is assumed that $f(t) = 0$ for $t < 0$, embedding medium is at rest prior to the instant $t = 0$ and the wave function is continuous function with continuous partial derivatives of the first and the second order. In view of problem symmetry, the wave equation (1) is usually rewritten in cylindrical coordinates ($r$, $\varphi$, $z$) noting that $\partial/\partial z = \partial/\partial\varphi = 0$. In order to naturally take into account the property of *causality*, the one-sided Laplace transformation, with respect to time $t$ is further applied on Eq. (1). This leads to the ordinary differential equation satisfied by modified Bessel functions of the first and the second kind $\mathrm{I}_n(\cdot),\mathrm{K}_n(\cdot)$, respectively. Here, $n$ denotes the order of the corresponding function. For the Laplace transformation parameter with the real and positive real part, the physically well-behaved solution is found as

$$\hat{u}(x,y,s) = \frac{\hat{f}(s)}{2\pi}\mathrm{K}_0(sr/c_0) \ , \tag{2}$$

here $r = (x^2+y^2)^{1/2} > 0$. In the majority of textbooks on electromagnetic wave theory, the solution is presented for the limiting case $s = \mathrm{j}\omega$, where j is imaginary unit and $\omega$ is real and positive angular frequency. Employing the relation (9.6.4) from [3], the frequency-domain solution is given as [1], [2]

$$\hat{u}(x,y,\mathrm{j}\omega) = \frac{\hat{f}(\mathrm{j}\omega)}{4\mathrm{j}} \mathrm{H}_0^{(2)}(\omega r/c_0) \ , \tag{3}$$

where $\mathrm{H}_0^{(2)}(\cdot)$ is zero order Hankel function of the second kind. Note that throughout the Kong's book [1], the time convention is $\exp(-\mathrm{j}\omega t)$ instead of $\exp(\mathrm{j}\omega t)$ applied here, which has the consequence of the different kind of the Hankel function. By our experience, students attending the lessons of basics of electromagnetic field theory can hardly imagine the properties of cylindrical functions. For that reason, we do not present the frequency-domain solution (3), but the solution in the original, space-time domain. To this end, we use the formulae (29.3.119) from [3] and transform (2) as

$$u(x,y,t) = f(t) \overset{(t)}{*} \frac{\mathrm{H}(t - r/c_0)}{(t^2 - r^2/c_0^2)^{1/2}} \, , \tag{4}$$

where * denotes the time-convolution, $\mathrm{H}(\cdot)$ is the Heaviside function. From the solution (4) is clearly seen that the corresponding two-dimensional Green's function is not impulsive, has an infinite duration and decays with time as $1/(t^2 - r^2/c_0^2)^{1/2}$. As a consequence, the solution is not only zero at the wave-front, but is non-zero everywhere within this wave front. A numerical example of the wave motion generated by the line source for the case of power-exponential excitation signature [5]

$$f(t) = f_{\max}(t/t_r)^{\nu} \exp[-\nu(t/t_r - 1)]\mathrm{H}(t) \, , \tag{5}$$

is given in Fig. 1. In Eq. 5, $f_{\max}$ is the pulse amplitude, $\nu$ is the rising exponent of the pulse and $t_r$ is the pulse rise time. The power-exponential pulse as defined by Eq. (5) is *causal* and *unipolar*. In the latter figure we plot the spatial distribution of the electric field strength at a certain instant. Since the pulse shape of electric field strength is, in fact, proportional to the time derivative of the scalar wave function $u(x,y,t)$, the resulting electric field distribution is of bipolar character (see Fig. 2a).

In the next example we observe what will happen in the case of collection of line sources placed along a line. This problem is shown for a finite number of line sources (Fig. 2b) and for a sheet source (Fig. 2c) that can be viewed as to be composed from an infinite number of line sources. In both cases, we can, except the tips of sources, observe unipolar plane-wave coming outward the source distribution. This phenomenon can be explained through Hadamard's method of steepest descent [4].. For the unbounded collection of line source, upon integrating the line sources along, for example, $x$-direction, we arrive at the response in the form

$$v(y,t) = \int_{x=-(c_0^2t^2-y^2)^{1/2}}^{(c_0^2t^2-y^2)^{1/2}} u(x,y,t)\mathrm{d}x = \frac{f(t)}{2} \overset{(t)}{*} \mathrm{H}(t - |y|/c_0) \, . \tag{6}$$

Again, the electric field is proportional to the time derivative of the corresponding wave function $v(y,t)$. But now, on the basis of the time-integration due to the presence of the time convolution in Eq. 6, the resulting response is unipolar as the excitation pulse was.

Further examples, as the diffraction of plane-wave on the dielectric obstacles (see Fig. 2d), are presented to students in purely illustrative form provided by post-processed data of COMSOL Multiphysics® solutions, because the mathematical and physical background of such phenomena in time domain is too complex and out of scope of the course.

## 3 Conclusion

We have presented new trends in the education of introductory courses on electromagnetic field theory as prepared in the Department of Radio Electronics in Brno. In this respect, the solutions of two-dimensional scalar wave equation are discussed in connection with the pulsed electromagnetic radiation from simple source distributions. The illustrative numerical examples are solved with the help of RF module in 2D and In-Plane modes for TE waves provided by COMSOL Multiphysics®.

The educational materials and models data are available at the web page of the course, so it's accessible to everyone.

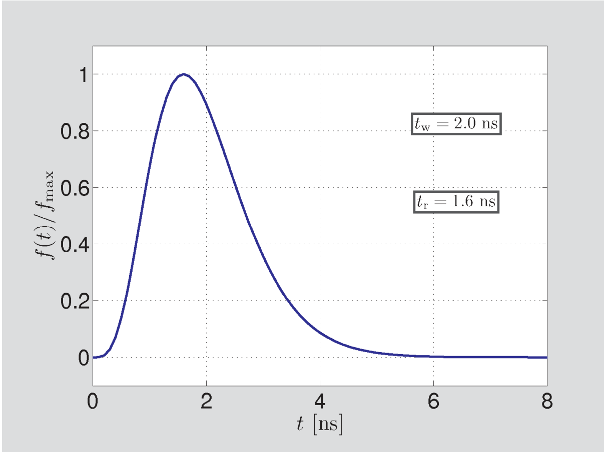


Figure 1: The example of the excitation pulse shape for ν = 2.

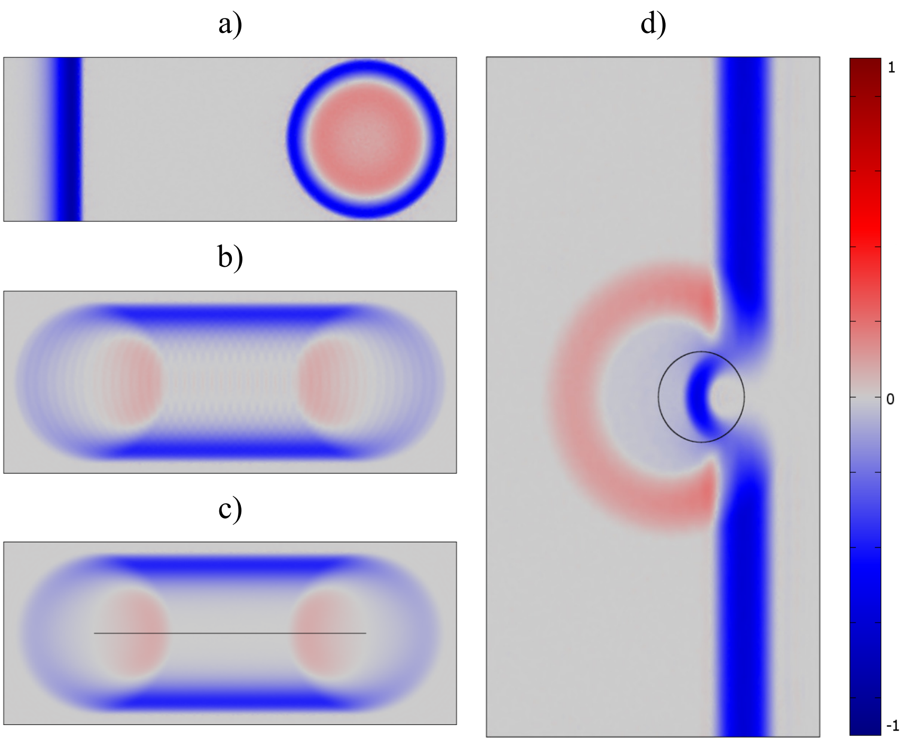


Figure 2: The example solutions of the electric field in $Vm^{-1}$: a) infinite sheet source generating a plane-wave and line source generating a cylindrical-wave; b) series of closely spaced line sources; c) finite sheet source; d) plane-wave passing through the dielectric cylinder

## References


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