MOBILE ROBOT DYNAMICS WITH FRICTION IN SIMULINK

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Abstract

This article focuses on the modeling of two-wheel mobile robot with differential chassis, which is moving in the plane. According to the application, it can be modeled with different accuracy levels depending on structural limits and other physical impacts. Main goal of this article is to include the effects of friction into the mobile robot dynamics. Four basic models of friction are combined into generalized model, which is then implemented as single Simulink block and added to model of mobile robot. Experiments show behaviour of friction models and mobile robot with friction.

1 Introduction

Advanced simulation and computing programs, such as Matlab/Simulink offer options to create and use a large variety of dynamic systems simulation models, that also includes the mobile robots. Based on the mathematical and physical relations, it is possible to obtain the mathematical models of mobile robots with different levels of accuracy, which can be used as basis for simulation models programming. They are useful during the robot prototype design process, as a parts of advanced control structures or as a simulation data source for physical variables, that cannot be measured on real models.

The main subject of interest of this article is the nonholonmic mobile robot with differentially driven wheels. In literature, this model of robot is usually considered as a mathematical model with simple dynamics [1], that includes the influences of robot's mass m and overall moment of inertia J to it's movement. The simulation model based on this mathematical model is usually used in more complex control structures and it is often extended by internal feedback control loop with PI controllers, designed to suppress the negative dynamics effects [1], [6]. The friction forces acting against robot movement are usually not taken into account in the mathematical model, because it is assumed, that the internal control loop can suppress them too, but simulation model itself without the internal control loop is not accurate in several views, for example:

- while there is a constant applied traction force, the robot accelerates without limit,
- wheel engines can set robot in motion at any non-zero force, that is not possible in reality,
- after robot achieves the constant velocity and has a zero acceleration, the traction forces are zero and the robot win never stop or slow down.

From this point of view, the measured simulation data for engine traction forces are not valid. To get simulation data, that are suitable for the feedforward neural model training [6], it is appropriate to increase the accuracy of mathematical model by adding influences of friction, that is usually not taken into account [1] or only simplified [6]. The main disadvantage of friction modelling is the non-linearity at zero velocity and subsequent numerical problems in calculations and simulations [3]. The focus of this article is in the implementation of friction into simulation model of nonholonomic mobile robot with differentially driven wheels, performed in the Simulink environment.

2 Mathematical model of mobile robot

Usually, the main control task for mentioned type of mobile robot is to control it's position in plane, which can be defined as a vector

$$\mathbf{q}_R = [x\ y\ \varphi\ \theta_R\ \theta_L]^T, \tag{1}$$

that describes the robot's centre of gravity CoG position in terms of the global coordinate system (GCS) by cartesian coordinates x, y on X and Y axes. The angle φ express the rotation of robot's local coordinate system against the GCS. Position vector can be extended by θ_R and θ_L for description of the right and left wheel angular positions. The overall robot's position in GCS is depicted on Fig. 1.

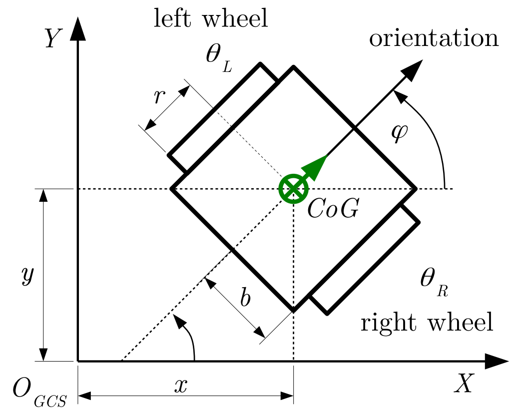

Figure 1: Definition of mobile robot dimensions and position in global coordinate system

The mathematical model of mobile robot can be interpreted as a conjunction of the kinematic and dynamic model. Kinematic model define the position of robot in plane [5] and the dynamic model covers the robot's dynamical properties and influences [1].

2.1 Kinematic model of mobile robot

Kinematic model of mobile robot with differentially driven wheels can be obtained from the assumptions for wheel movement in the plane [4]:

- the robot can move in plane while its Z axis is always perpendicular to this plane,
- wheels are rolling without forward slip fulfilling the holonomic constraints for movement of this type of robot in plane [2],
- it is assumed, that in a small time samples, even while the robot is moving straight forward, it always moves on an arc, which centre is the *ICC* - instantaneous centre of curvature and this arc's radius is in term of CoG defined as R_o [5],
- the robot base is solid, it's mass is constant and the wheels are only moving parts of the robot.

The kinematic model can be derived from the geometric properties of the robot motion in plane by defining the angular velocity of mobile robot centre of gravity CoG in respect to *ICC* as [5]

$$\omega(t) = \frac{v_R(t)}{R_o + b} \quad \text{or} \quad \omega(t) = \frac{v_L(t)}{R_o - b}, \qquad \omega(t) = \frac{v_R(t) - v_L(t)}{2b}, \tag{2}$$

where b expresses the distance between wheel and CoG, which lies in the middle of wheels rotation axis. The radius R_o of an arc than can be defined as

$$R_o = \frac{b[v_R(t) + v_L(t)]}{-v_R(t) + v_L(t)} \tag{3}$$

and the linear velocity $v(t)$ of mobile robot moving at this arc is

$$v(t) = R_o\omega(t) = \frac{v_R(t) + v_L(t)}{2}. \tag{4}$$

In terms of GCS, the kinematic equations for mobile robot movement defined for linear velocity $v(t)$ and its overall angular velocit $\omega(t)$ can be summed as

$$\begin{aligned} \dot{x}(t) &= v(t)\cos\varphi(t), \\ \dot{y}(t) &= v(t)\sin\varphi(t), \\ \dot{\varphi}(t) &= \omega(t). \end{aligned} \tag{5}$$

If necessary, the kinematic model (5) can be also defined for the wheel linear velocities $v_R(t)$, $v_L(t)$ from angular velocity equations (2) and for the wheel angular velocities ω_R, ω_L by incorporating the wheel radius r [1]. For the purposes of mobile robot control, the $x(t)$ and $y(t)$ position coordinates of robot's CoG are the most important, the orientation angle $\varphi(t)$ can be also used as one goal of the control. The angular positions of wheels $\theta_R(t)$, $\theta_L(t)$ can be obtained as well [2], but they have rather informational value. Kinematic model defined as system of equations (5) is the ideal mobile robot mathematical model, which is sufficient for some types of applications, but if the goal is to get more realistic data, it has to be extended by dynamic effects.

2.2 Dynamic model of mobile robot

Given that the dynamic properties of mobile robot have an important influence to its motion, mainly from the wheel engines traction forces F_R, F_L, it is possible to incorporate them into the mobile robot's dynamic model. According to the second Newtons law [1]

$$F_i = m_i a_i, \tag{6}$$

the linear acceleration of mobile robot's base can be interpreted as a derivation of its linear velocity v and interpreted as a sum

$$\dot{v} = \frac{F_R}{m} + \frac{F_L}{m}. \tag{7}$$

while the traction forces F_R, F_L can be defined from the robot's overall torque

$$\tau_{robot} = \tau_R + \tau_L, \tag{8}$$

where τ_R, τ_L express partial wheel torques. Those torques can be defined in terms of the traction forces as

$$\tau_R = F_R b \quad \text{and} \quad \tau_L = -F_L b, \tag{9}$$

while the left engine traction force F_L is considered in opposite direction to suit the relation (8) for the robot's overall torque τ_{robot}, this situation is depicted on Fig. 2

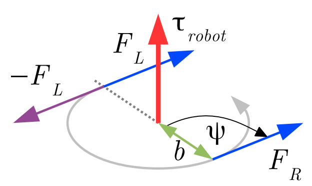

Figure 2: Definition of mobile robot's overall torque τ_{robot} and orientation of traction forces F_R, F_L

Angular acceleration of mobile robot $\dot{\omega}$ can be expressed by its torque τ_{robot} and robot's overall moment of inertia J as

$$\tau_{robot} = J\dot{\omega}. \tag{10}$$

By substituting the partial wheel torques (9) for right and left engine τ_R, τ_L into the sum (8) of total mobile robot's torque τ_{robot} and by comparing it with the equation (10), it is possible to obtain the angular velocity $\dot{\omega}$ of two wheeled mobile robot (11) as a function of a robot's engines traction forces.

$$\dot{\omega} = F_R\frac{b}{J} - F_L\frac{b}{J}. \tag{11}$$

Mathematical model of mobile robot with differentially driven wheels, that consist of a kinematic (5) and dynamic (7), (11) model may be represented by block diagram depicted on Fig. 3 and it can be programmed as a simulation model in the Simulink environment.

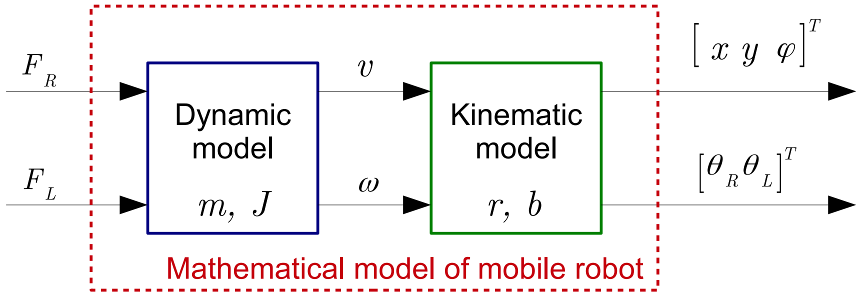

Figure 3: Block diagram of a mobile robot with the wheel engines traction forces as inputs and the position coordinates as outputs

The following part of the article analyses the friction as an extension to the mobile robot dynamic model expressed by (7) and (11).

2.3 Friction in mobile robot dynamics

Friction acting against the movement is dependent on the object's mass by the normal force F_N and it can be classified as a static or a kinetic friction [3], [7]. Static friction is considered only at zero velocity and it is represented by its coefficient μ_s as a force threshold F_s, which must be overcome to set the object, in this case a mobile robot, moving. Until reaching this force threshold, the effect of wheel traction force F_{ext} is compensated by static friction force F_{fr}. After overcoming the F_s, the robot is set in motion, now affected by kinetic friction, which is usually represented as a constant Coulomb friction, defined as

$$F_{fr}(v) = \mu_k F_N \cdot \mathrm{sign}(v), \tag{12}$$

where μ_k is the Coulomb friction coefficient. This (12) kinetic friction model defines the friction force at non-zero velocities, it is independent on the size of contact area and always acts against the movement, however the friction force F_{fr} is dependent on a moving object's velocity direction, no matter the size. In general, the coefficients are defined in term $\mu_s \geq \mu_k$, while in the case of $\mu_s > \mu_k$ it is possible to consider the effect of stiction [3], illustrated on Fig. 4.

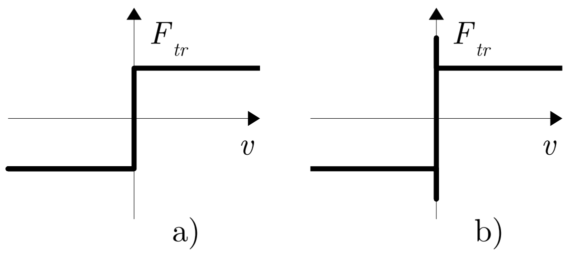

Figure 4: Friction models: a) Coulomb friction model, b) Coulomb friction model with stiction effect

Kinetic model of friction can be further extended by considering the viscous friction, defined by μ_v coefficent, which causes an increase in the friction force with increasing velocity v of moving object. Finally, at small velocities and with the active stiction effect, it is possible to apply the Stribeck curve defined by coefficent a_{st}, which replaces the step change of the friction force from F_s to F_k by step-less change at a small velocities interval [3]. By combining the above mentioned friction models, a generalized model of friction can be obtained as [3]

$$F_{fr}(F_{ext}, v) = \begin{cases} F_{ext} & v = 0 \quad a \quad F_{ext} < F_s \\ F_s & v = 0 \quad a \quad F_{ext} \geq F_s \\ F_k + (F_s - F_k)e^{-a_{st}v} + F_v v & v > 0 \\ -F_k + (F_k - F_s)e^{a_{st}v} - F_v v & v < 0 \end{cases} \tag{13}$$

and it can be represented as a function of velocity v depicted in Fig. 5.

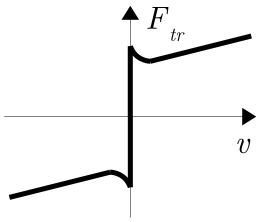

Figure 5: Generalized model of friction - a function of velocity v

In Simulink environment, it is advantageous to create a variable block, that allows user to select desired friction model through the dynamic mask. Fig. 6 shows the programmed block with different settings for typical friction models [7]. The block have an additional state input v_{sta} and reset output v_{rst}, both used later when the block is integrated into the simulation model of mobile robot dynamic model in the following part of article.

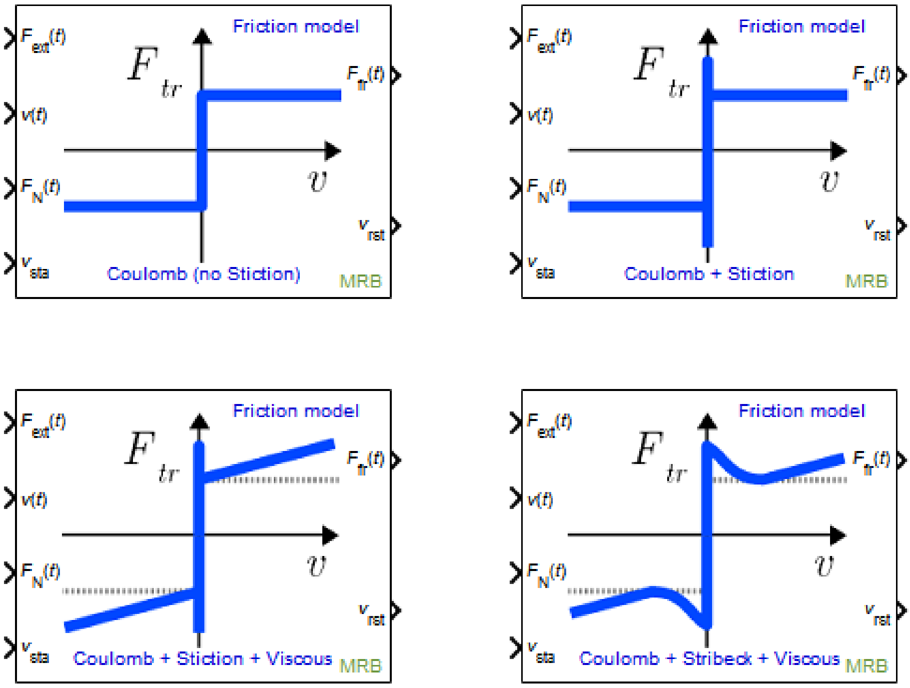

Figure 6: Different setting for created friction model in Simulink

3 Mobile robot dynamics with friction in Simulink

Dynamic model of the mobile robot, shown in Fig 3, that covers the influences of robot's mass m and inertia J defined for linear (7) and angular acceleration (11) can be extended in the Simulink environment by connecting the generalized friction (13) block. Since it may be expressed for external, traction force

F_{ext} and linear velocity v, it can be integrated into the dynamic model of mobile robot individually for both wheels, while it is necessary to express their linear velocities v_R, v_L as a functions of the traction forces F_R, F_L.

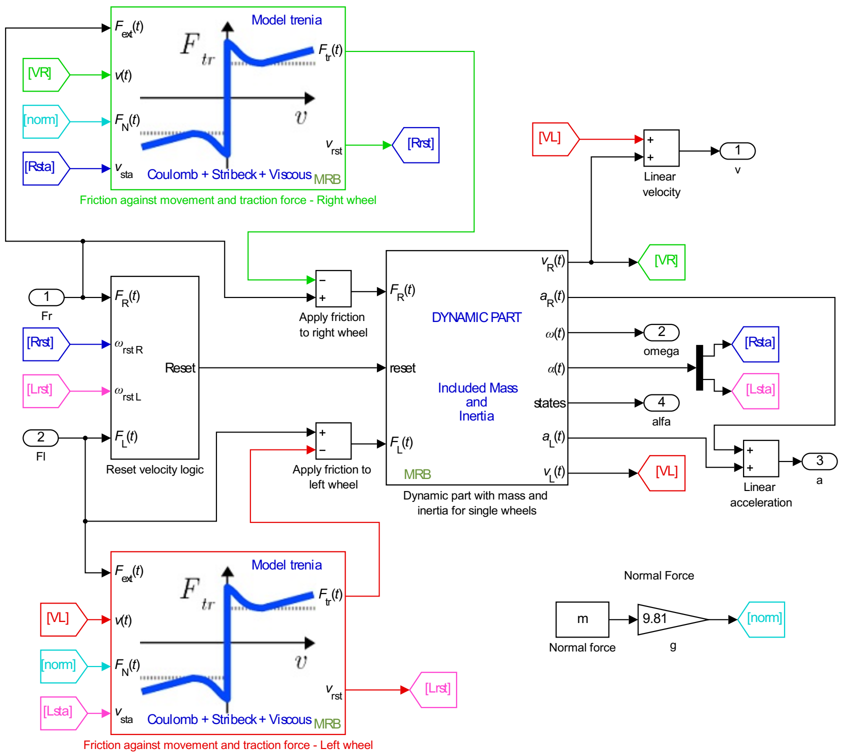

Figure 7: Dynamic model of the mobile robot extended by adjustable friction

The state outputs from integrator blocks, that calculates the wheel linear velocities are connected to the friction block's ports v_{sta} and those integrators have an external reset inputs, connected to output ports v_{rst}. The implementation of this scheme in Simulink environment is shown in Fig. 7.

4 Experiments and results

The first simulation experiment, which output is depicted on Fig. 8 illustrates the effects only of the Coulomb friction due to acting external force and velocity of the moving object. The inputs of friction force block are the normal force $F_N = 10$ N, the external force F_{ext} is simulated as a sine wave with an amplitude of 1, the linear velocity v is the same sine signal reduced to half and delayed by 1 second, output of the block is the frictional force F_{fr} dependent on inputs and friction coefficients relevant for rolling movement.

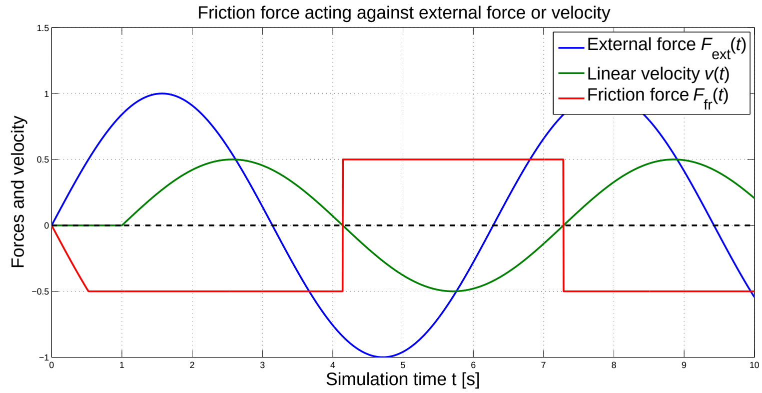

Figure 8: The Coulomb friction - effects of static and kinetic friction with regard to the external force and caused linear velocity

In the second experiment, plotted on the Fig. 9, a generalised model of friction (13) with same input conditions is used. From the behaviour of the friction force F_{fr}, the influences of stiction, Coulomb kinetic friction, viscous friction and the stribeck curve at small velocities can be seen.

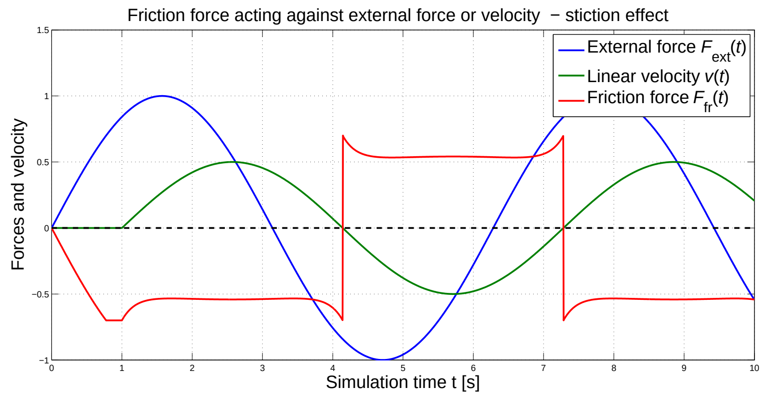

Figure 9: The generalized friction - the effects of static and kinetic friction with regard to the external force and caused linear velocity

The generalized model of friction (13) is also considered in the next experiment: a constant traction forces $F_R = 0.5$N a $F_L = 0.51$N are fed for 3 seconds to the simulation model of mobile robot inputs. The mobile robot block in experiment shown on Fig. 10 implements the block diagram from Fig. 3.

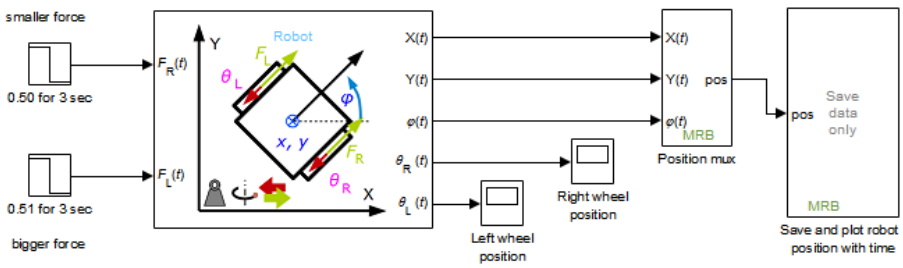

Figure 10: The simulation experiment for demonstration of the friction effects acting on mobile robot movement

Both forces are greater than the threshold of static friction force F_s and therefore, the robot is immediately set in motion. At the moment, when the robot starts moving and its velocity v slowly rises, the friction force F_{fr} gradually decline according to stribeck curve. After reaching the local minimum at constant Coulomb friction, with rising velocity the viscous friction impact is more significant. After period of 3 seconds, the excitation forces are no longer in effect and because of friction, the robot starts to slow down until it stops - the linear velocity v is than zero as well as friction forces. Fig. 11 plots the overall linear velocity of robot $v(t)$ and the friction forces acting on both wheels.

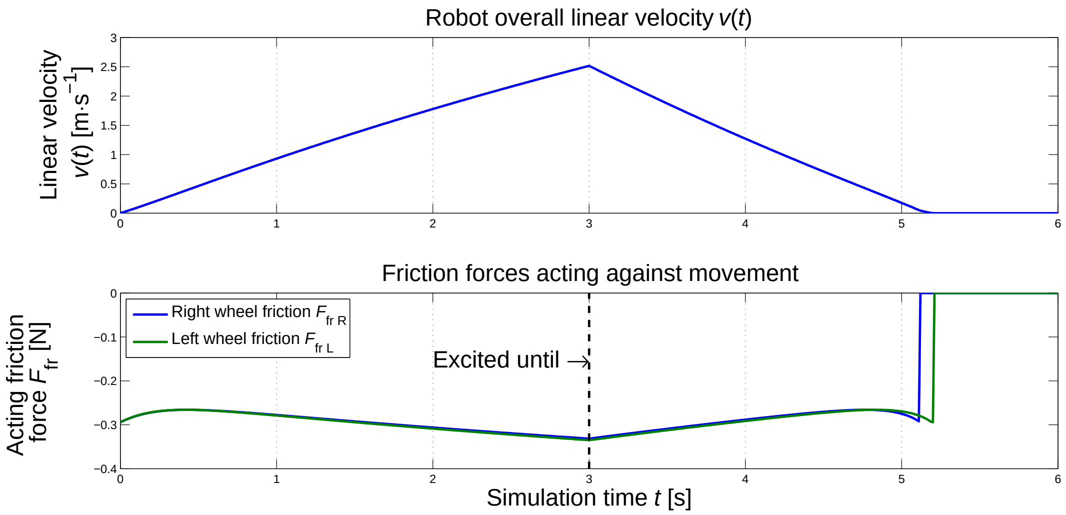

Figure 11: The simulation experiment - the overall linear velocity and the impact of friction forces to the robot wheels

Given that the input excitation forces F_R and F_L are not equal, the robot starts to turn in meaning of a weaker force, in this case it turns right. Mobile robot's movement in the plane for last simulation experiment is shown on Fig. 12.

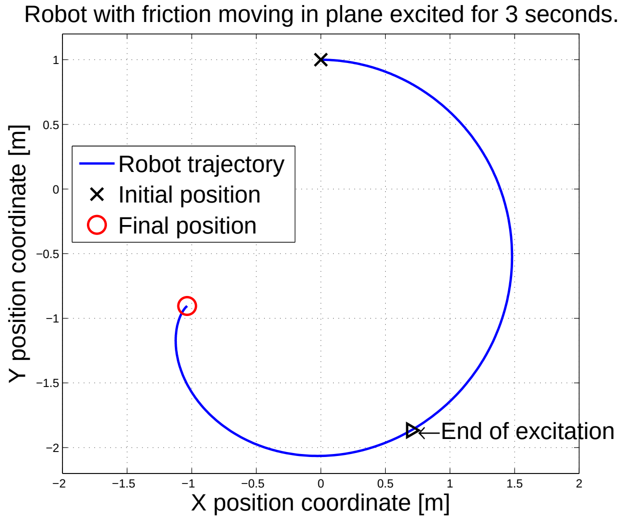

Figure 12: The simulation experiment - robot movement in the plane affected by friction

The Simulink environment enables to program the simulation model of mobile robot with dynamics based on mathematical model and use it in simulation experiments to obtain desired data. It also offers options to create a simulation block, that realizes a the generalized model of friction and implement it into a dynamic model of the mobile robot.

5 Conclusion

This article describes how to include the friction into the dynamic model of the mobile robot and thus suppress the inaccuracies mentioned in the introduction of the article. If the friction is included in the dynamics of the simulation model of robot, then

- the robot does not move until the engines have a sufficient force to overcome the F_s threshold,
- due to friction, an already moving robot without the external input will slow down and stop,
- the viscous friction ensure that robot has a maximal velocity,
- at a constant velocity, the engines develop a traction forces that keeps the robot moving.

By taking the friction into account, the simulation model of mobile robot with differentially driven wheels is more complex, however the obtained data are more accurate and they are usable for example, to train the feed-forward neural model [6].

To control the mobile robot position or for the trajectory tracking [1], [2], the reference values are usually angular velocities of wheels ω_R a ω_L. For these reason, the mobile robot shown in Fig. 3 has to be extended by the internal feed-back control loop with PI controllers, that can suppress the dynamic effects. Block diagram of the proposed control structure is shown on Fig. 13.

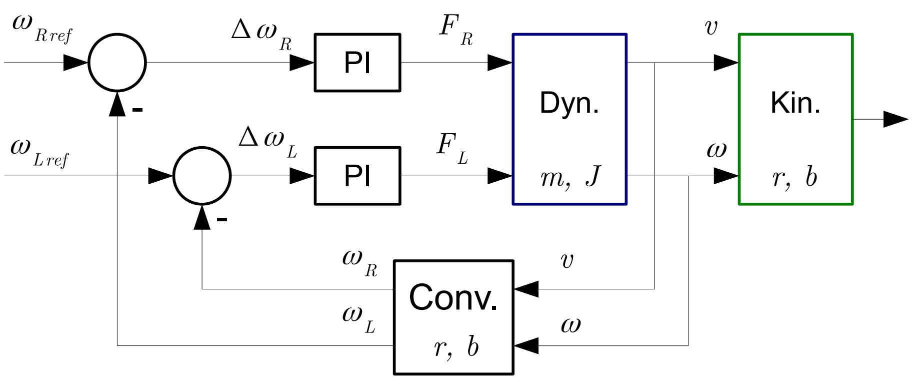

Figure 13: Diagram of a mobile robot with an internal control loop

Thanks to the internal control loop, the influences of friction should be suppressed like other robot dynamics, but obtained simulation data will be more realistic.

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