PROGRAM FOR CALCULATING THE COEFFICIENTS OF THE DIFFERENTIAL EQUATION USING THE METHOD OF GRADUAL INTEGRATION

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Abstract

Nowadays, the identification of systems plays an important role in the control of systems. This paper is aim to a description of the method of gradual integration and its use for calculating the coefficients of the differential equation describing the regulated system. This paper also describes program for calculating the coefficients of the differential equation, which describes regulated system.

1. Principles the method of gradual integration

Identification of dynamic systems is a mature field of research and a wide variety of methods has been reported, oriented to both linear and nonlinear systems. Identification techniques can be classified into parametric and non-parametric. [1]
Nonparametric methods - return models that are described by a table or a graph, such as the impulse response or the frequency response.
Parametric methods - return models that can completely describe the system behavior using a finite number of parameters (typically relative small), such as transfer function models and state-space models. Parametric models are estimated from either time domain data or frequency domain data. [1]

The method of gradual integration is parametric and deterministic method. Input signal must by deterministic and system must be without interference. This method can´t be used when system is ordinary operation, because there is many interference signal. We can use this method, when we make a first identification of the system and we use clearly defined input signals. [2]

The principle of this method is the gradual integration of the input signal and the response output signal from the system. [3] If we assume a system, which can be described by differential equations, which has a form:

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \ldots + a_1 y' + a_0 y = x \tag{1}$$

Where x is the change of the input variable, y is the change of the output variable of the regulated system and a_0 - a_n are coefficients of the differential equation.

If the conditions in equation (2) and (3) are valid, for time $t < 0$ and $t \to \infty$, then the system and output variable in this time are in the steady state.

$$y'(0) = y''(0) = \ldots = y^{(n-1)}(0) = 0 \tag{2}$$

$$y'(\infty) = y''(\infty) = \ldots = y^{(n-1)}(\infty) = 0 \tag{3}$$

If input and output signals fulfill conditions in equation (4) and (5), then we integrate equation (1) from $t=0$ to $t \to \infty$, and apply conditions in equation (2), (3), (4) and (5), then we can determine coefficient a_0 as in equation (6).

$$x(0) = x(\infty) \tag{4}$$

$$y(0) = y(\infty) \tag{5}$$

$$a_0 = \frac{\int_0^\infty x.dt}{\int_0^\infty y.dt} \tag{6}$$

If we want to determine coefficient a_1, then first we must integrate equation (1), from t to infinite equation (7), then from 0 to infinite - equation (8) and apply conditions in equation (2),(3),(4) and (5)

$$-a_n y^{(n-1)} - a_{n-1} y^{(n-2)} - \ldots - a_1 y + a_0 \int_t^\infty y.dt = \int_t^\infty x.dt \tag{7}$$

$$\int_0^\infty -a_1 y.dt + \int_0^\infty \int_t^\infty a_0 y.dt^2 = \int_0^\infty \int_t^\infty x.dt^2 \tag{8}$$

Equation (9) shows execution of coefficient a_1. If we can integrate equation (1) from t to infinite again, then we calculate coefficient a_2. When we repeat this process, we can determine every coefficient differential equation.

$$a_1 = \frac{1}{\int_0^\infty y.dt} . \left(a_0 \int_0^\infty \int_t^\infty y.dt^2 - \int_0^\infty \int_t^\infty x.dt^2 \right) \tag{9}$$

For coefficient a_υ, where $\nu > 0$, we can derive general functionality, which is shown in equation (10),

$$a_\nu = \frac{(-1)^\nu}{\underset{1}{S}(y)} \left[\underset{\nu+1}{S}(x) - \sum_{i=0}^{\nu-1} (-1)^i . a_i . \underset{\nu+1-i}{S}(y) \right] \tag{10}$$

In this case $\underset{\nu}{S}(y)$ signify a υ-multiple integration of variable y - equation (11)

$$S(y) = \int_0^\infty \int_t^\infty \int_t^\infty \ldots \int_t^\infty y.dt^\nu \tag{11}$$

All coefficients in the differential equation for stable system must be positive. Therefore we calculate coefficient until one of the coefficients is negative.

Figure 1 shows graphical interpretation of the method of gradual integration for pulsed output signal.

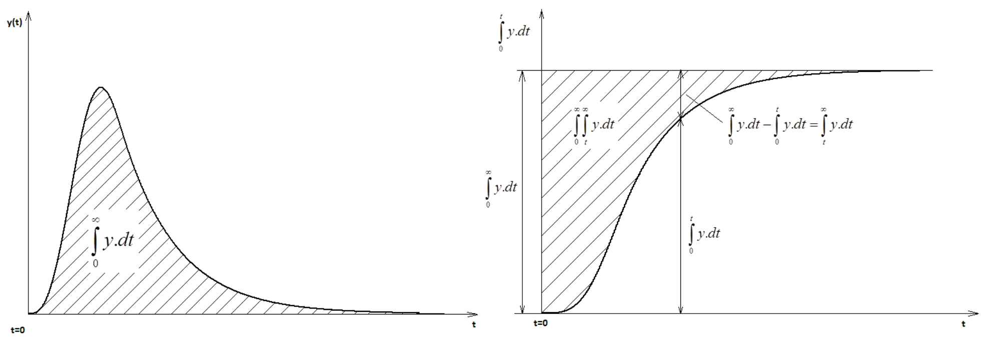

Figure 1: Graphical interpretation of the method of gradual integration

This process of identifying coefficient of the differential equation can be used for a special case of waveforms input variable. Example for change input variable of regulated system with step waveform with defined value as: x (∞)-x (0) we can determine coefficient a_0 as.

$$a_0 = \frac{x(\infty) - x(0)}{y(\infty) - y(0)} \tag{12}$$

For coefficient a_ν, where $\nu > 0$, we can derive general functionality, which is shown in equation (13)

$$a_\nu = \frac{1}{y(\infty) - y(0)} \sum_{i=0}^{\nu-1} (-1)^{\nu+i+1} . a_i . \underset{\nu-i}{S} \left[y(\infty) - y \right] \tag{13}$$

If waveform of input variable has a periodic course, where have same course for positive values and for negative values. Then integral from 0 to infinite of this waveform is $\int_0^\infty x.dt = 0$. Example of this waveform is in the figure 2.

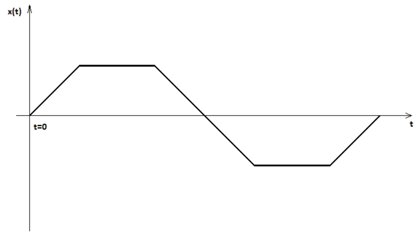

Figure 2: Example of waveform of periodic course of input variable

With method of gradual integration we can determine coefficient a_0 as equation (14):

$$a_0 = \frac{\int_0^\infty \int_t^\infty x.dt^2}{\int_0^\infty \int_t^\infty y.dt^2} \tag{14}$$

For coefficient a_ν, where $\nu > 0$, we can derive general functionality, which is shown in equation (15)

$$a_\nu = \frac{(-1)^\nu}{\underset{2}{S}(y)} \left[\underset{\nu+2}{S}(x) - \sum_{i=0}^{\nu-1} (-1)^i . a_i . \underset{\nu+2-i}{S}(y) \right] \tag{15}$$

We can use this method for another deterministic input signal too. This method is good realizable with computer technic, where integral of waveform of input and output signal are calculate with numerical mathematic method, example as trapezoidal method, Simpson's method and another.

2. Program for calculating coefficients

For calculate coefficients of the differential equation was made graphic user interface in matlab, which is show to figure 3. This program calculates integral of input and output signal with trapezoidal numerical method.

This GUI is consists of main window, which can be divide to three parts. First part is used for enter parameters of input signal, as is kind of signal, start and final value of input signal, period or time duration. In this part define waveform of input signal too. We can define waveform of input signal by time dependence or table, where are time intervals and values of input signal in this time intervals. In second part is for enter parameters and define waveform too, but for response output signal. We can draw input and output waveform in graphs. In third part are buttons for start calculate coefficients, for reset every parameters and for end of program. [4]

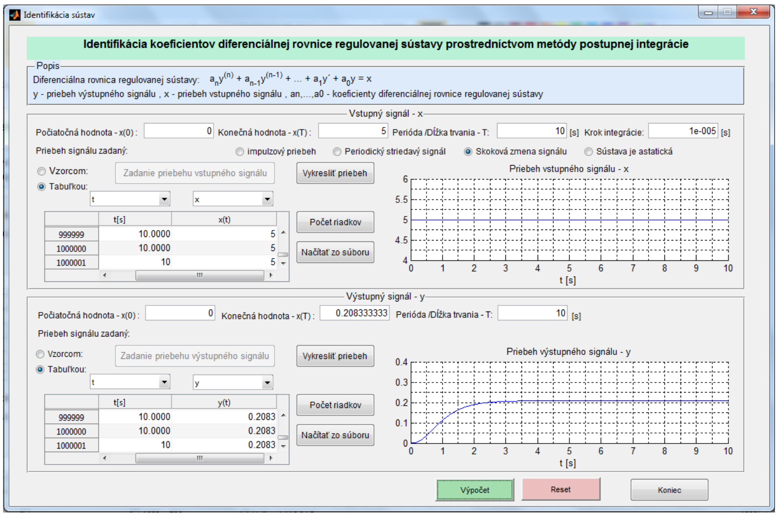

Figure 3: Program for calculating the differential equations using the gradual integration method.

From the specified parameters, program selects the right algorithm for calculating a coefficient. Then start process of calculating integrals of the waveform of input and output signals. Program use trapezoidal numerical method, for to calculate integrals. Next program determine the appropriate coefficient, and if it is positive, program continue in determining next coefficient, but it is negative program terminate a process of calculating coefficient and displays coefficients in new window. [4]

The figure 4 shows calculated coefficients of the differential equation for step input signal and his response output signal. Both signals are specified by values in table. The calculated coefficients are quite accurately calculated, but coefficients a_3 and a_4 have a very small error.

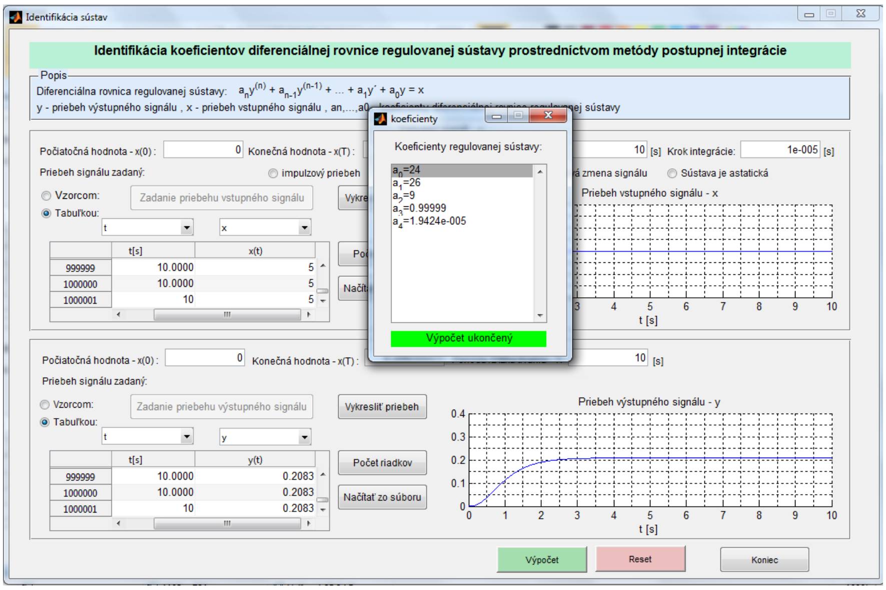

Figure 4: The calculated coefficients of differential equation of the regulated system.

Conclusion

In this paper, the method of the gradual integration has been presented. The method of gradual integration is deterministic method, which is useful determine same parameters as is amplification, time constants and is useful for first setting PID regulators. In this paper developed program for calculating coefficient of the differential equation has been presented, too. The calculated coefficients with this application are calculated acurately, but higher has a small error. This errror is caused by method, used for mathematical integration, integration step size, accurancy of the specified parameters and accurancy of the values of the input and output waveforms.

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