GENERALIZED PREDICTIVE CONTROL OF THERMO-OPTICAL PLANT

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Abstract

In this paper method of predictive control is addressed, proposed and tested. Controller design uses model of a process, which replicates real laboratory process behaviour. The proposed model is tested in model based predictive control of thermo-optical plant. This paper deals with theoretical and practical methodology, offering approach for control design and its successful application.

1 Introduction

Predictive control has become popular over the past twenty years as a powerful tool in feedback control for solving many problems for which other control approaches have been proved to be inefficient. Predictive control is a control strategy that is based on the prediction of the plant output over the extended horizon in the future, which enables the controller to predict future changes of the measurement signal and to base control actions on the prediction, see [4].

Model-based predictive control (MBPC) is a particular class of optimal control. The first main advantage of Model predictive control (MPC) is that constraints (due to: manipulated variables physical limitations, operating procedures or safety reasons) may be explicitly specified into this formulation. The second main advantage of MPC is its ability to be used for both simple and complex model based processes (time delays, inverse responses, significant nonlinearities, multivariable interaction, modeling uncertainties).

Concept of MBPC has been heralded as one of the most significant control developments in the recent ten years. MBPC algorithms were originally developed for linear processes, but the basic idea can be transferred to nonlinear systems too. MPC shows improved performance because the process model allows current computations to consider future dynamic events.

The main idea of MPC is the output signal prediction at each sampling point in time [3]. The prediction is implicit or explicit, depending on the model of the process to be controlled. In the next step a control signal value is selected with a purpose of bringing the predicted process output signal back to the reference signal by minimizing the area (criteria function) between the reference and output signal.

The paper is organized as follows. First, generalized predictive control (GPC) algorithm and the design of model are discussed in Section 2. The thermo-optical plant is briefly introduced in Section 3. The reliability and effectiveness of the presented predictive method is shown on the application in Section 4. Summary and conclusions are given in Section 5.

2 Generalized Predictive Control

The most common predictive control law is the very well known Generalized Predictive Control (GPC) algorithm. The GPC method has become one of the most popular MPC methods both in the industry and in the field of science. It has been successfully implemented in many industrial applications, showing good performance and certain degree of robustness.

Generalized Predictive Control has many ideas in common with the other predictive controllers since it is based on the same concepts, but differs in some aspects too.

A particular MPC controller is the Generalized Predictive Controller (GPC), proposed by [1], which has become one of the most widespread controllers in the MPC.

The mathematical model of nonlinear plant is based on observed input - output data.

We have linearized the nonlinear process applying ARX model.

The GPC algorithm uses a Controlled Autoregressive and Integrated Moving Average (CARIMA) model of the SISO systems, see [2]:

$$\bar{A}(z^{-1})y(k) = B(z^{-1})u(k-1) + \frac{C(z^{-1})}{\Delta}\xi(k) \tag{1}$$

where $\Delta=1\text{-}z^{-1}$, $C(z^{-1})$ is colouring filter, $\bar{A}(z^{-1})$ is n ordered polynomial denominator with coefficients a_i and $B(z^{-1})$ is m ordered polynomial numerator with coefficients b_i, $u(k)$ is an input, $y(k)$ is an output of dynamic system and $\xi(k)$ is a system output error or a noise of output measurement.

If $\xi(k)=0$, ARX model can be expressed ($\Delta\bar{A}\,(z^{-1})=A(z^{-1})$):

$$A(z^{-1})y(k) = B(z^{-1})\Delta u(k-1) \tag{2}$$

Parameters of the ARX model structure are estimated using the least-squares method.

And the transfer function is (n=2, m=2)

$$\frac{Y(z^{-1})}{\Delta U(z^{-1})} = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + (a_1 - 1)z^{-1} + (a_2 - a_1)z^{-2} - a_2 z^{-3}} \tag{3}$$

In the most cases, difference between system outputs and reference trajectory is considered in combination with a cost function on the control effort. A general objective function is the following quadratic form, see [3]

$$\min J(\Delta u) = \min\{\sum_{j=N_1}^{N_2}[r(k+j) - \hat{y}(k+j\,|\,k)]^2\gamma_j + \sum_{j=1}^{N_u}\Delta u(k+j-1\,|\,k)^2\lambda_j \tag{4}$$

Here, r is the desired set point, γ_j and λ_j are weighting sequences, determining the relative importance of the different terms in the cost function, u and Δu are the control signal and its increment, respectively. Parameter N_2 is the maximum of prediction horizon, N_1 is the minimum of prediction horizon and N_u is the length of control horizon. Output predicted by the model is $\hat{y}(k)$. In this context, $x(k+j|k)$ is the predicted value of $x(k+j)$ calculated for instant k.

The optimal solution $\Delta\boldsymbol{u}$ (only the first element is applied to the plant) is then given by minimizing the objective function defined by Eq. (4). Assuming, that there are no constraints, calculation leads to the following control law:

$$\Delta\boldsymbol{u}=(\boldsymbol{G}^{\boldsymbol{T}}\,\Gamma\boldsymbol{G}+\,\Lambda\boldsymbol{I})^{-1}\boldsymbol{G}^{\boldsymbol{T}}\,\Gamma(\boldsymbol{r}\text{-}\boldsymbol{y}_{\boldsymbol{f}}) \tag{5}$$

Matrix G consists of the impulse response parameters of $B(z^{-1})/A(z^{-1})$, r is the vector with the coefficients of the future set-point and y_f is the free response, Γ and Λ are the weighting matrices with coefficients γ_j and λ_j, respectively.

To obtain the closed loop characteristic equation, the control structure can always be expressed in a classical LTI form, as shown in Fig. 1, where R and S are polynomials in z^{-1}.

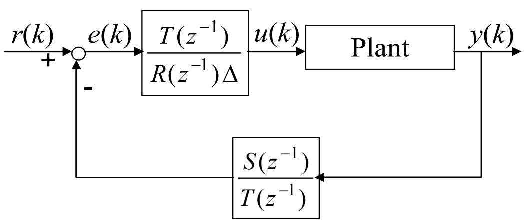

Figure 1: MPC structure

The characteristic equation of this general closed loop system is:

$$\delta_0(z^{-1}) = R(z^{-1})A(z^{-1}) + B(z^{-1})S(z^{-1}) \tag{6}$$

where $A(z^{-1})=\Delta\bar{A}(z^{-1})$.

From Fig. 1, it is seen, that:

$$R(z^{-1})\Delta u(k) = T(z^{-1})r(k) - S(z^{-1})y(k) \tag{7}$$

Denoting:

$$K=(\boldsymbol{G}^{\boldsymbol{T}}\,\Gamma\boldsymbol{G}+\Lambda\boldsymbol{I})^{-1}\boldsymbol{G}^{\boldsymbol{T}}\,\Gamma \tag{8}$$

Then the expressions of polynomials are:

$$R(z^{-1})=\frac{T(z^{-1})+z^{-1}\sum_{i=N_1}^{N_2}k_{1i}H_i(z^{-1})}{\sum_{i=N_1}^{N_2}k_{1i}z^i}$$
$$S(z^{-1})=\frac{\sum_{i=N_1}^{N_2}k_{1i}F_i(z^{-1})}{\sum_{i=N_1}^{N_2}k_{1i}z^i} \tag{9}$$

where polynomials $H_i(z^{-1})$ and $F_i(z^{-1})$ are derived of the free response, k_1 is the first row of the matrix K from Eq. (8).

3 Case Study

The thermo-optical plant [5] is a simple laboratory physical model of the thermo dynamical and optical system called DIGICON USB thermo-optical plant (Fig. 2).

Figure 2: Thermo-optical plant

Its thermal channel consists of one heater represented by an electric bulb and one cooler represented by a small fan. The output of this channel is the temperature inside the tube. Measurement of the output value is performed by a thermal sensor. The second dynamics is represented by the optical channel. Within this channel it is possible to generate light by a LED and measure the intensity of the light by photo resistor. The optical channel is even more comfortable for conducting experiments because the time constants are much smaller compared to the thermal channel. The base of the model covers also the electronic part. This part includes one connector for input (voltage) and two others connectors for data communication. One of these is used for communication with the data acquisition card AD512 and another one is the USB port that can be connected directly to the computer (instead of using an expensive data acquisition card). The front panel of the base has five information LEDs. The body of the electronic part is equipped with integrated circuits for communication and signal conversion.

The thermo-optical plant was controlled by Matlab. The dynamics of this plant is not so fast, therefore it is not necessary to have a special data acquisition card to perform control algorithms. So the thermo-optical plant can be connected to the USB port of any PC or notebook where Matlab is running. In our experiments intensity of the light using the photo resistor was measured. The input to the model is voltage (0-5 V). The I/O characteristic of the thermo-optical plant is shown in Fig. 3.

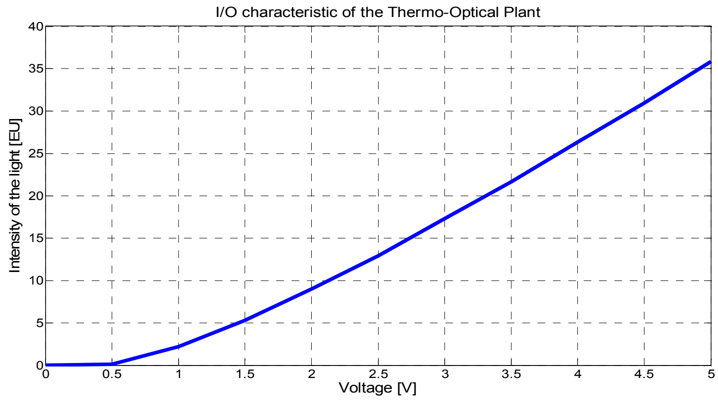

Figure 3: I/O characteristic of the thermo-optical plant

From the I/O characteristic, the working area was chosen for the inputs <3.1; 3.35> and the outputs <17.7; 20.2>.

We have linearized the thermo-optical plant with sampling period T_s=0.1654s and discrete transfer function of the model is

$$G(z) = \frac{B(z^{-1})}{\overline{A}(z^{-1})} = \frac{4.326z^{-1}}{1-0.2635z^{-1}} \tag{10}$$

The discrete transfer function according (3) is obtained

$$\frac{Y(z^{-1})}{\Delta U(z^{-1})} = \frac{4.326z^{-1}}{1-1.2635z^{-1}+0.2635z^{-2}} \tag{11}$$

4 Simulation Results

For this case the parameters for predictive control are N_2=10, N_1=1, N_u=5, γ_j=1, λ_j=2.

Fig. 4 shows time responses of the controlled, reference and manipulated variables under GPC – without setpoint prediction. Fig. 5 shows time responses of the controlled, reference and manipulated variables under GPC – with setpoint prediction.

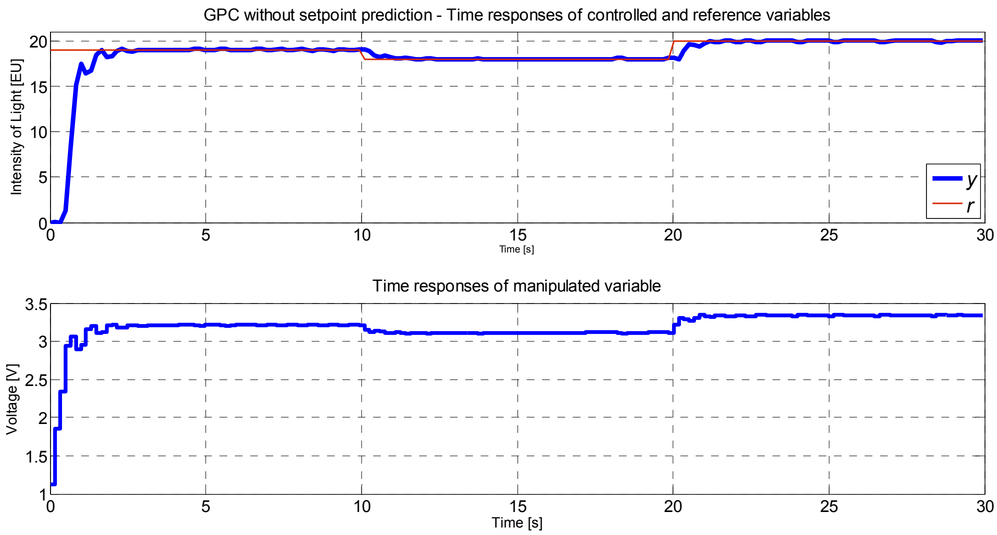

Figure 4: Time responses of the variables under GPC (without setpoint prediction)

For the robust stability analysis, it is necessary to calculate every possible combination of active constraints to obtain every possible characteristic equation. This will be the aim of our research in the future.

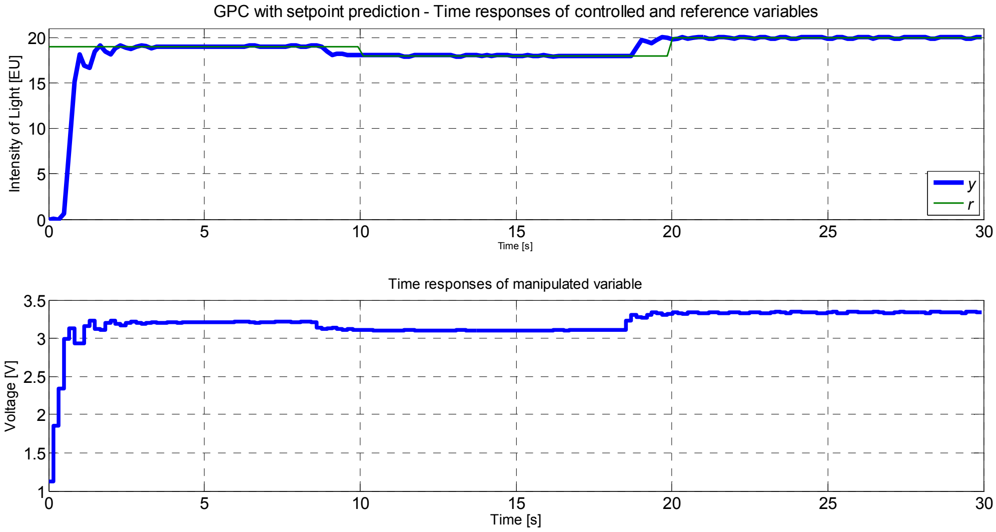

Figure 5: Time responses of the variables under GPC (with setpoint prediction)

5 Conclusion

In this paper generalized predictive controller for nonlinear process had been designed and verified on case study of a laboratory thermo dynamical system.

Simulation example illustrates the potential offered by the predictive control - GPC.

In our future research we will be dealing with stability of predictive control methods.

Acknowledgements

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