IMPULSE SWITCHING FUNCTIONS OF POWER INVERTERS IN MATLAB ENVIRONMENT

Pavol Štefanec, Branislav Dobrucký, Viliam Jaroš

Department of mechatronics and electronics, Faculty of Electrical Engineering,
University of Žilina

Abstract

Among many methods of power converters analyses - as classical analytic ones, Laplace transform or Fourier analyses – the most suitable for both dynamical- and steady state operation is transient analysis uses dynamic state space modelling or Z-transform method. Impulse switching functions (ISF) is one of the fastest methods for dynamic system solving. Because of strongly non-harmonic output quantities of power converters systems the power series of time pulses it deals with. Simulation results using MATLAB environment show possible consideration of impulse switching functions for equivalent scheme of the single phase voltage inverter system with resistive – inductive load.

1 Introduction

There are many methods of power converters analyses. Classical analytic methods, Laplace transform or Fourier analysis is suitable mainly for steady state operation [1], [2]. Transient analysis uses dynamic state space modeling or Z-transform method. One of the fastest methods is with uses impulse switching functions (ISF). For power converters systems ISF are strongly non-harmonic. Then it deals with power series of time pulses. Fig. 1 shows possible impulse switching function of considered single phase voltage inverter system and equivalent scheme of this system with resistive – inductive load.

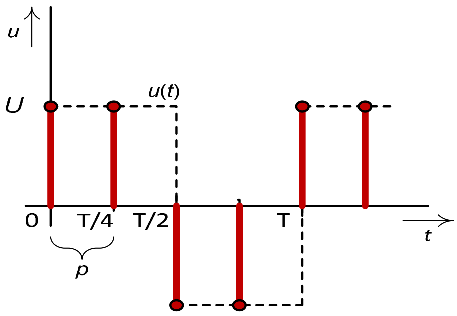

Figure 1: Impulse switching function of single phase voltage inverter system

2 Impulse switching function using Z-transform

Using basic definition of Z-transform - and taking into account Z-images of constant- and alternating series [3] we can write

$$U(z) = U\frac{z^2+z}{z^2+1} \tag{1}$$

where $U(z)$ is output voltage in z-plain and roots of polynomial of denominator are

$$z_{1,2} = \pm \mathrm{j} = \pm(-1)^{\frac{1}{2}} = e^{\pm\frac{1}{2}} \tag{2}$$

placed on boundary of stability in unit circle [3], [4].

Applying inverse Z-transform for converter output phase voltages in Z-domain we can create impulse switching functions. For inverse Z-transform $U(z) \leftrightarrow \{u_n\}$ one can use the residua theorem [4], [5]

$$\sum_{i=1}^{N} \underset{z=z_i}{res} U(z) z^{n-1} = \sum_{i=1}^{N} \lim_{z=z_i} (z-z_i) U(z) z^{n-1} \tag{3}$$

where n = 0, 1, 2, …, N is number of poles.

Then

$$U(z) \leftrightarrow \{u_n\} = \sum_{i=1}^{N} \lim_{z=z_i} (z-z_i) U \frac{z+1}{(z+1)(z-1)} z^n \quad . \tag{4}$$

After adapting

$$u_n = u\left(n\frac{T}{4}\right) = U\sqrt{2}\sin\left(n\frac{\pi}{2}+\frac{\pi}{4}\right). \tag{5}$$

Taking discrete state-space model for single-phase converter output current as state-variable considering 1st order load (resistive-inductive)

$$x_{n+1} = \mathbf{F}_{T/4} x_n + \mathbf{G}_{T/4} \left\{ u\left(n\frac{T}{4}\right) \right\}, \tag{6}$$

where $\mathbf{F}_{T/4}, \mathbf{G}_{T/4}$ are fundamental and transition matrices of system parameters.

Applying Z-transform and considering resistive-inductive load (the matrices have only one element)

$$z\,X(z) = F_{T/4} X(z) + G_{T/4} Y_\infty(z) \rightarrow X(z) = X_\infty G_{T/4} \frac{z^2+z}{\left(z-F_{T/4}\right)(z^2+1)}, \tag{7}$$

where $X(z)$ is image of founded state-variable (inductor current or capacitor voltage), and $Y_\infty(z)$ is maximum of steady state value (U/R or U, respectively).

And after adaption and simplification

$$\begin{aligned} X(z) &= X_\infty G_{T/4} \frac{z(z+1)}{\left(z-F_{T/4}\right)(z+1)(z-1)} = \\ &= X_\infty G_{T/4} \frac{z}{\left(z-F_{T/4}\right)(z-1)} = X_\infty G_{T/4} \frac{z}{(z-z_0)(z-z_1)}, \end{aligned} \tag{8}$$

where $z_0 = F_{T/4}, z_1 = 1$ are polynomial roots of denominator $X(z)$.

Applying inverse Z-transform

$$x\left(n\frac{T}{4}\right) \equiv \{x_n\} = X_\infty G_{T/4} \lim_{z\to z_i} \left\{ \sum_{i=0}^{1} (z-z_i) \frac{z_i}{(z_i-z_0)(z_i-z_1)} z_i^{n-1} \right\} \tag{9}$$

Finally we get discrete form of state variable (converter output state variable – inductor currents or capacitor voltages)

$$x\left(n\frac{T}{4}\right) \equiv \{x_n\} = X_\infty G_{T/4} \left(\frac{1}{F_{T/4}-1} F_{T/4}^{\;n} + \frac{1}{1-F_{T/4}} 1^n \right) = X_\infty \frac{G_{T/4}}{1-F_{T/4}} \left(1^n - F_{T/4}^{\;n}\right) \tag{10}$$

For calculation of function values is necessary to determine the $F_{T/4}, G_{T/4}$ function values which are the state-variable values in ¼ -instant of time period (i.e. they are state- and transition responses). These can be obtained e.g. by using recursive relation for one-pulse solution:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = A \cdot x(t) + B \cdot Y_\infty(t) \tag{11}$$

thus recursive relation

$$x(k+1) = F_\Delta x_k{}' + G_\Delta X_\infty, \tag{12}$$

where $i_{k=0} = I_0 = 0$. Solution in z-domain yields

$$X(z) = X_\infty \frac{G_\Delta (1 - r z^{-1})^{60}}{(z - F_\Delta)(1 - r z^{-1})}, \tag{13}$$

where F_Δ, G_Δ are discrete impulse responses of state-variables gained by some of identification methods. The second fraction term is z-image of the partial sum of voltage impulses (1÷60) [3], since $r^{an} \cdot z^{-n};\ a < 0$ is geometrical series, see next Fig. 2.

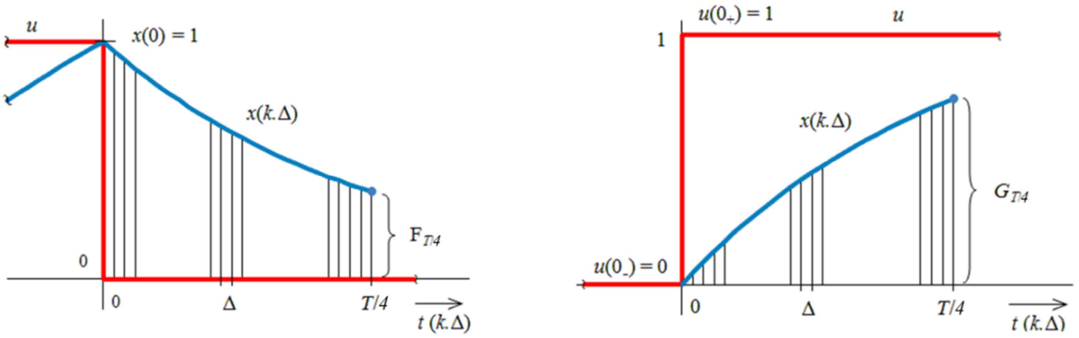

Figure 2: To determination of $F_{T/4}$, $G_{T/4}$

After choosing $\Delta = T/360$, k will be the in the range of 0÷59, thus $F_{T/4} = y(60)$, and $F_{T/4} = 1 - G_{T/4}$. Supposing time constant of the load equal to T/2 and

$$F_\Delta = 0.9944598;\ G_\Delta = 0.0055401; \tag{14}$$

those values of $F_{T/4}, G_{T/4}$ will be

$$F_{T/4} = F_\Delta q^{N-1} = F_\Delta{}^{60} = 0.71652923, \tag{15}$$

because of $q = F_\Delta$, and

$$G_{T/4} = G_\Delta \frac{1 - F_\Delta{}^{60}}{1 - F_\Delta} = 1 - F_\Delta{}^{60} = 0.2834707. \tag{16}$$

Now, one can calculate state-variable $x\left(n\frac{T}{4}\right)$ for any n, Fig. 3.

It is also possible to change the step of series e.g. for step equal $T/2$, by determining of $F_{T/2}$ and $G_{T/2}$

$$F_{T/2} = F_{T/4}{}^2 = 0.5134141, \tag{17}$$

and regarding to $G_{T/2}$:

$$x\left(\frac{T}{4}\right)=F_{T/4}\,x(0)+G_{T/4}\,X_{\infty} \tag{18}$$

It is also possible to change the step of series e.g. for step equal $T/2$, by determining of $F_{T/2}$ and $G_{T/2}$

$$F_{T/2}=F_{T/4}^{\;2}=0.5134141, \tag{19}$$

and regarding to $G_{T/2}$:

$$x\left(\frac{T}{4}\right)=F_{T/4}\,x(0)+G_{T/4}\,X_{\infty}, \tag{20}$$

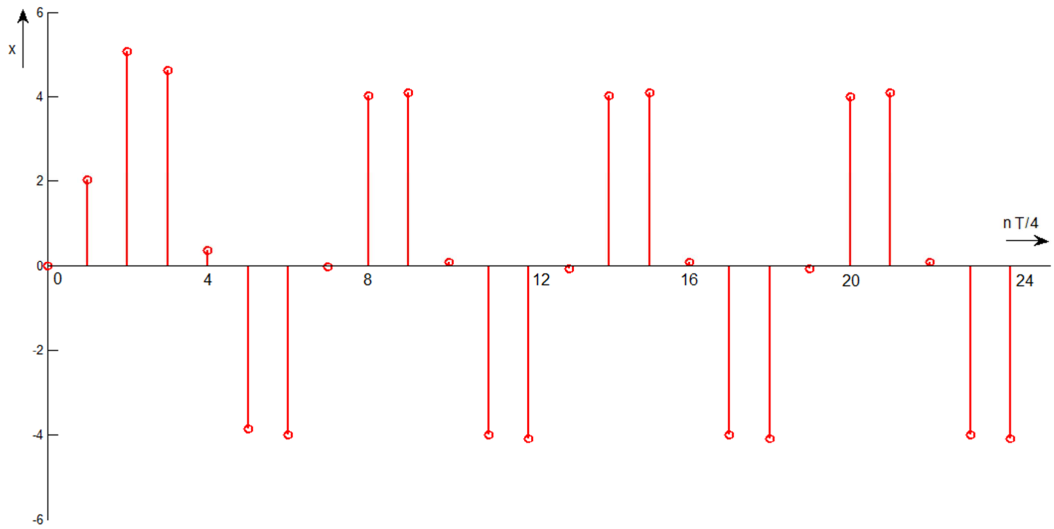

Figure 3: Discrete response sequence of state-variable $x\left(n\frac{T}{4}\right)$ of single-phase inverter

$$x\left(\frac{T}{2}\right)=F_{T/4}\,x\left(\frac{T}{4}\right)+G_{T/4}\,X_{\infty}\rightarrow G_{T/2}=0.4865857. \tag{21}$$

So, then one can calculate

$$x_{n+1}=F_{T/2}\,x_n+G_{T/2}\left\{u\left(n\frac{T}{2}\right)\right\} \tag{22}$$

Where

$$u\left(n\frac{T}{2}\right)=U(-1)^n\ . \tag{23}$$

The steady state value of ‘periodical’ sequence $x\left(n\frac{T}{4}\right)$ can be determined, based on condition

$x(0)=-x\left(\frac{T}{2}\right)$ or vice versa, i.e. $x\left(\frac{T}{2}\right)=-x(0)$.

Then

$$x\left(\frac{T}{2}\right)=F_{T/2}\left[-x\left(\frac{T}{2}\right)\right]+G_{T/2}\,X_{\infty}\rightarrow x\left(\frac{T}{2}\right)=\frac{G_{T/2}}{1+F_{T/2}}X_{\infty}=0.3215153\,X_{\infty}, \tag{24}$$

so $x(0) = X_0 = -0.3215153\,X_{\infty}$.

Settings this value into $\{x_n\}$ or $\{x_{n+1}\}=\ldots$ we get

$X_0 = -0.3215153\ (n=0);$

$X_{T/4} = 0.0530956\ (n=1);$

$X_{T/2} = 0.513845\ (n=2);$

Based on zero order hold function [3] and total mathematical induction one finally yields

$$u(k\Delta) = U_{DC}\sqrt{2}\sin\left[\text{integer}\left(\frac{6}{T}k\Delta\right)\frac{\pi}{2}+\frac{\pi}{4}\right], \tag{25}$$

where *´integer´* means integer function, T is time period and $k\Delta$ is discretizing time.
Derived relations for voltages can be used for state variable calculations in electrical engineering systems.

$$x(k+1) = F_\Delta x_k + G_\Delta u_{\alpha,\beta}(k\Delta), \tag{26}$$

where F_Δ a G_Δ are discrete impulse responses of state-variables (see above).

3 Impulse switching function used for three-phase power converter systems

Similar method is possible to use for three-phase system. As a conclusion it can be shown the use ISFs for applying for three-phase α,β-system [6]

$$u_\alpha\left(n\frac{T}{6}\right) = U_{\text{DC}}\frac{2}{3}\sin\left(n\frac{\pi}{3}+\frac{\pi}{6}\right) \tag{27}$$

$$u_\beta\left(n\frac{T}{6}\right) = -U_{\text{DC}}\frac{2}{3}\cos\left(n\frac{\pi}{3}+\frac{\pi}{6}\right). \tag{28}$$

Final relation for discrete form of state variable /converter output state variable – inductor currents or capacitor voltages will be

$$x\left(n\frac{T}{6}\right) \equiv \{x_n\} = X_\infty \frac{G_{T/6}\left(1+F_{T/6}\right)}{F^2_{T/6}-F_{T/6}+1}.\left[F^n_{T/6}+\sqrt{3}\frac{1-F_{T/6}}{1+F_{T/6}}\sin\left(n\frac{\pi}{3}\right)-\cos\left(n\frac{\pi}{3}\right)\right] \tag{29}$$

where $n = 0,1,2,..\infty$; $F_{T/6}, G_{T/6}$ are fundamental and transition matrices (in general) of system parameters and X_∞ is exciting function value ($U/_R$ or U, respectively).

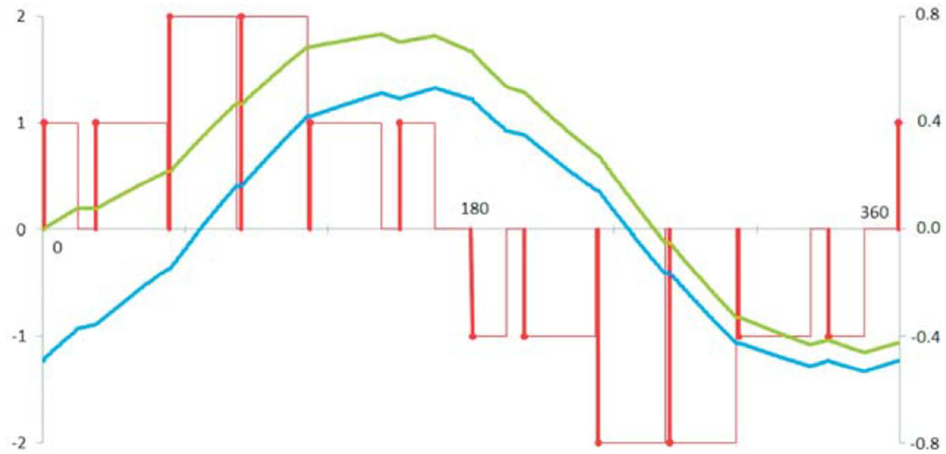

Figure 4: State variable response under sine PWM supply [6]
Legend: red – impulse switching function, blue – transient response, green – steady state waveform, pink –switching pulses generated by inverter

As an example, transient- and steady-state waveforms of state variable under sine PWM supply with minimum number of switching states (12-pulse switching voltage function) are shown in figure.

Acknowledgement

The authors wish to thank for the financial support to R&D operational program Centre of excellence of power electronics systems and materials for their components No. OPVaV-2008/2.1/01-SORO, ITMS 26220120046.

References

[1] ZASKALICKA, M., ZASKALICKY, P., BENOVA, M., ABDALMULA, M. A. R., DOBRUCKY, B.: Analysis of Complex Time Function of Converter Output Quantities Using Complex Fourier Transform/Series. Communications - Scientific Letters of the University of Zilina, No. 1(12), pp. 23-30, 2010.

[2] DOBRUCKY, B., MARCOKOVA, M., POKORNY, M., SUL, R.: Using orthogonal and discrete transform for single-phase PES transients - a new approach. In: Proceedings of the 27th IASTED Int'l Conference, Feb. 11-13, 2008, Innsbruck, Austria, Acta Press, 2008, ISBN 978-0-88986-633-1, pp. 60-65.

[3] VICH, R.: Z-transformation and some of its Utilization (in Czech), SNTL Mathematical Seminar, Prague, 1983.

[4] ARAMOVIC, J. G., LUNC, G. L., ELSGOLC, L. C.: Functions of Complex Variable, Operator Calculus, Stability Theory (in Russian – translated into Slovak), Alfa, Bratislava, 1973.

[5] MORAVCIK, J.: Mathematical Analysis 3 (in Slovak), Textbook, Alfa Publishing House, Bratislava, 1992, ISBN 80-05-01044-3

[6] DOBRUCKY, Branislav, POKORNY, Michal, BENOVA, Mariana, ABDALMULA, Mahmud Ali Rzig, Modeling of power converters using Z-transform. Communication – Scientific Letters of the University of Zilina, 2013, vol. 15, no. 3, pp. 43-47.

Pavol Štefanec
email: pavol.stefanec@fel.uniza.sk

Branislav Dobrucký
email: branislav.dobrucky@fel.uniza.sk

Viliam Jaroš
email: viliam.jaros@fel.uniza.sk