M-FILES FOR QUARTIC SPLINES

J. Kobza,   P. Ženčák

Dept. MA&AM , Faculty of Sciences, UP Olomouc

Abstract

The overwiev of M-files worked out for computing local parameters of quartic splines interpolating function values, mean values or derivative values is presented . Local representations used for computing function values of such splines in arbitrary point and corresponding M-files are described. M-files for computing local parameters and function values of mean values smoothing splines and some shape (monotonicity, convexity) preserving quartic smoothing splines complete the overwiev.

1 Notation

Splines have been recognized as usefull tool in interpolation and approximation problems. We can find them to be implemented in widely used software products for scientific computing and computer graphics (Mathematica, Maple, Matlab). The widely known collection of Fortran programs published in [1] enables the user to work with PP-representation in case of cubic function values interpolating and smoothing splines (no mean or derivative values are explicitly mentioned at all). Splines of all another degrees are handled here basically in B-spline basis (but they can be turned to PP basis for plotting and another purposes). This package of programs was (with some inovations) used as the basis for Spline Toolbox in MATLAB (de Boor, 1990-95, see [2]).
In applications the splines of degrees 1-6 are the most frequently used. Our idea for several last years was to work out in details the proper local representations for function, mean and derivative values interpolation and smoothing with quadratic and quartic splines. They can be used then for corresponding formulation of continuity conditions, initial or boundary conditions - all in terms of local parameters with known geometric meaning to the user. This article develops and extends the overwiev given in [10]. The M-files worked out for linear and quadratic splines in 1D and 2D are described in [15]. In this contribution we present the collection of 21 M-files for quartic splines.

The notation used here corresponds with the notation used in the articles cited; in M-files the Matlab programming rules result in some differences (e.g.

for notation in indexes etc.).
Let us have a vector of spline knots $\mathbf{x} = \{x_i,\ i = 0(1)n+1\}$,

$$a = x_0 < x_1 < \ldots < x_n < x_{n+1} = b, \quad \textit{with stepsizes} \qquad h_i = x_{i+1} - x_i$$

and let $d \in \{1,2,3,4\}$. *The quartic spline* $s_{4d}(x)$ *with the defect* d *on the knot sequence* $\mathbf{x}$ *is a piecewise polynomial (PP) function with the properties:*

1. $s_{4d}(x)$ *is a fourth degree polynomial on every interval* $[x_i, x_{i+1}]$, $i = 0(1)n$ *;*
2. $s_{4d} \in C^{4-d}[x_0, x_{n+1}]$ *(Continuity Condition – CC) .*

In major part of this work quartic splines with defect one are treated (the splines with defect two will be mentioned in sections dealing with quasilocal or shape preserving splines). For brevity we shall use notation $s(x) = s_{41}(x)$.
Let us have given values g_i, $i = 0(1)n$ *and let* $x_i < t_i < x_{i+1}$ *for* $i = 0(1)n$
(or g_i *are given in* $x_i = t_i$, $i = 0(1)n+1$ *- simple grid).*
We say that the quartic spline $s_{4d}(x)$ *solves the problem of*

- *function values interpolation (FVI) if* $s(t_i) = g_i$ *;*
- *mean values interpolation (MVI) if* $\int_{x_i}^{x_{i+1}} s(x)dx = h_i g_i$ *;*
- *derivative interpolation (DVI) if* $s'(t_i) = g_i$ *.*

We shall denote in the following
$s_i = s(x_i)$ the spline function values in knots ,
$m_i = s'(x_i)$ the spline derivative values in knots,
$M_i = s''(x_i)$ the second derivative values in knots.

A quartic spline is not determined by interpolatory conditions uniquelly, some free parameters (3 on simple grid, 4 on general grid) can be used (usually the boundary conditions).

2 Quartic splines on simple (rectangular) grid

We shall follow firstly the special case of FVI when the points of interpolation coincide with spline knots .

2.1 Quartic splines on simple 1D grid

Problem:
Let us have given knots $\mathbf{x} = \{x_i,\ i = 0(1)n+1\}$ *and function values* $\mathbf{s} = \{s_i,\ i = 0(1)n+1\}$. *Find a quartic spline* s_{41} *interpolating function values* $\mathbf{s}$ *in knots* $\mathbf{x}$.

The needed local parameters **m, M** we can compute using M-file **s41dmM**. For computing spline local parameters on frequently used equidistant knotset $\mathbf{x}$ the more simple M-file **lp1ds4e** based on recurrences for the local parameters m_i only (see [6], p.64) can be used. Spline function value $s(t)$ can be computed from local parameters by M-file **s4gval**.

Example: The FVI spline (Fig. 1) on simple grid can be computed using following sequence of function calls:

```
x=[0.1:0.1:2]; s=sin(log(x)).*sin(2*pi*x); plot(x,s,'o'); hold on;
[m,M]=s41dmm(x,s,[]);
xx=[0.1:0.01:2]; yy=zeros(size(xx));
for i=1:length(yy) yy(i)=s4gval(xx(i),x,s,m,M); end; plot(xx,yy);
```

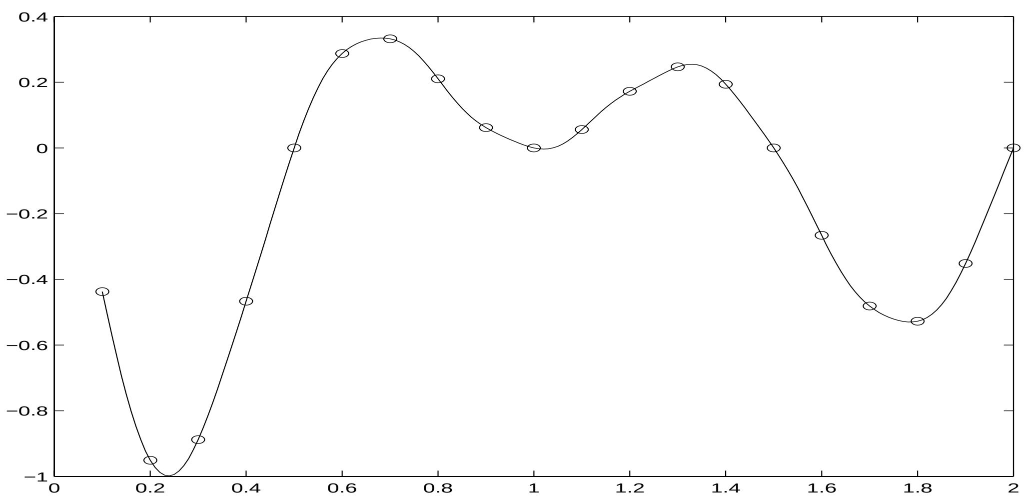

Figure 1: The solution of FVI problem (solid line) on simple grid (the circle)

2.2 Biquartic splines on simple grid

Problem:
Let us have given knots grid $\mathbf{x} \times \mathbf{y}$ *with* $\mathbf{x} = \{x_i,\ i = 0(1)n+1\}$, $\mathbf{y} = \{y_j,\ j = 0(1)m+1\}$ *and function values* $\mathbf{S} = \{s_{ij},\ i = 0(1)n+1, \jmath = 0(1)m+1\}$. *Find a biquartic spline with defect one interpolating function values* $\mathbf{S}$ *in knots* $\mathbf{x} \times \mathbf{y}$.

In case of equidistant knots in vectors **x, y** we can use the M-file **lps42d** for computing local parameters. Spline function value $s(u, v)$ can be computed from local parameters using M-file **s42deval**.

Example: The FVI spline on rectangular grid (Fig. 3) can be computed using following sequence of function calls:

```
hx=pi/3; x=[-pi:hx:pi]; y=x; hy=hx; S=sin(X.^2+Y.^2);
S10=zeros(size(S));S01=S10;S20=S10;S02=S10;
S11=S10;S21=S10;S12=S01;S22=S10;
[S10,S01,S20,S11,S02,S21,S12,S22]=...
  lps42d(S,S10,S01,S20,S11,S02,S21,S12,S22,x,hx,y,hy);
xx=[-pi:pi/30:pi]; yy=xx;
for i=1:lengt(xx)
 for j=1:length(yy)
  sxy(i,j)=s42deval([xx(i),yy(j)],x,y,hx,hy,S,S10,S01,S11,...
    S20,S02,S21,S12,S22);
 end;
end;
[XX,YY]=meshgrid(xx,yy); surf(XX,YY,sxy);
```

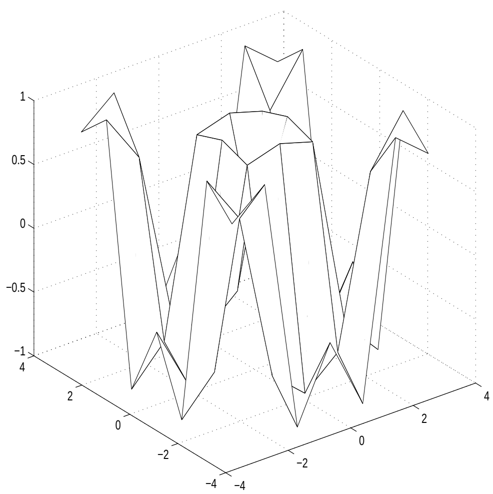

Figure 2: The data on rectangular grid

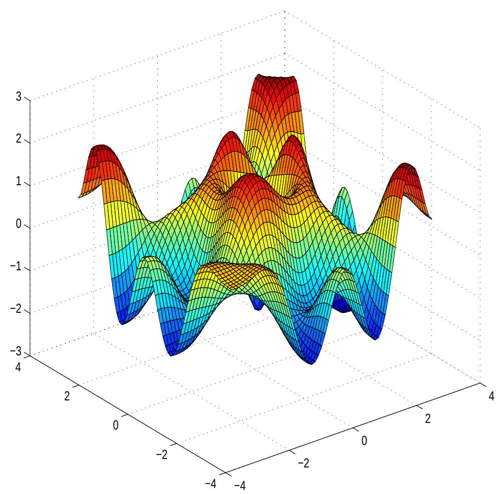

Figure 3: The solution of FVI problem on simple rectangular grid

3 Quartic splines on general knotset

We can use various local representations of quartic splines according to boundary conditions used or another needs. Some overwiev of local representations for FVI, MVI and DVI quartic splines is given in [8].

3.1 FVI problem in various local representations

Problem:
Let us have given knots $\mathbf{x} = \{x_i,\ i = 0(1)n+1\}$ *and points* $\mathbf{t} = \{t_i,\ i = 0(1)n\}$

such that $x_i < t_i < x_{i+1},\ i = 0(1)n$, *and function values* $\mathbf{g} = \{g_i,\ i = 0(1)n\}$. *Find a quartic spline* s_{41} *with knots* $\mathbf{x}$ *interpolating function values* $\mathbf{g}$ *in points* $\mathbf{t}$.

The needed local parameters **m, M** of **(g,m,M)** representation we can compute using M-file **spl4**.The computing of function values using these local parameters is implemented in function **spl4hodn**.
If the **(g,s,m)** representation is used, the local parmeters **s, m** are computed by M-file **s41smfvi** . Then spline function value $s(x)$ can be computed from these local parameters by M-file **s4gsmval** .

3.2 DVI problem in (g, s, M) representation

Problem:
Let us have given knots $\mathbf{x} = \{x_i,\ i = 0(1)n+1\}$ *and points* $\mathbf{t} = \{t_i,\ i = 0(1)n\}$ *such that* $x_i < t_i < x_{i+1},\ i = 0(1)n$, *and values of the first derivatives* $\mathbf{g} = \{g_i,\ i = 0(1)n\}$. *Find a quartic spline* s_{41} *with knots* $\mathbf{x}$ *interpolating values of derivatives* $\mathbf{g}$ *in points* $\mathbf{t}$.

The needed local parameters **s, M** we can compute using M-file **spl4d**. The computing of function values using these local parameters is implemented in function **spl4hodd**.

Example: The DVI spline on general grid (Fig. 4) can be computed using following sequence of function calls:

```
x=[-2:0.5:5]; t=[-1.8:0.5:4.85]; g=exp(t).*(sin(2*t)+2*cos(2*t));
conds=[0,1]; valconds=[0.1,-0.1;-80,-329];
[s,M,h,d]=spl4d(t,x,g,conds,valconds);
plot(x,s,'x'); hold on;
for i=1:length(t) y(i)=spl4hodd(t(i),s,M,x,g,t); end;
plot(t,y,'o'); xx=[-2:0.01:5]; yy=zeros(size(xx));
for i=1:length(yy) yy(i)=spl4hodd(xx(i),s,M,x,g,t); end;
plot(xx,yy);
```

3.3 MVI problem in various representations

Problem:
Let us have given knots $\mathbf{x} = \{x_i,\ i = 0(1)n+1\}$ *and mean values* $\mathbf{g} = \{g_i,\ i = 0(1)n\}$. *Find a quartic spline* s_{41} *interpolating mean values* $\mathbf{g}$.

The unknown local parameters **m, M** of **(g,m,M)** representation we can compute using M-file **spl4m**. The computing of function values using these local parameters is implemented in function **spl4hodn**. For the special case of equidistant knots the M-file **s4mvise** can be used for computing local parameters **s, m** of **(g,m,M)** representation. Then spline function value $s(x)$ can be computed from these local parameters again by M-file **s4gsmval** .

Example: The MVI spline (Fig. 5) can be computed using following sequence of function calls:

```
x=[-3:0.2:3]; g=diff(atan(x))./diff(x);
stairs(x,[g,g(length(g))]); hold on;
conds=[0,1]; valconds=[0.1,-0.06;0.1,0.06];
[m,M,h]=spl4m(x,g,conds,valconds);
stairs(x,[g,g(length(g))]); hold on;
xx=[-3:0.01:3]; yy=zeros(size(xx));
for i=1:length(yy) yy(i)=spl4hodn(xx(i),m,M,x,g); end;
plot(xx,yy);
```

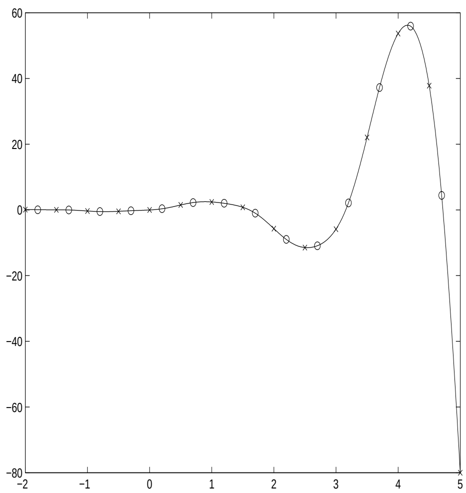

Figure 4: The solution of DVI problem with (solid) different knots (x-mark) and points of interpolation (circle)

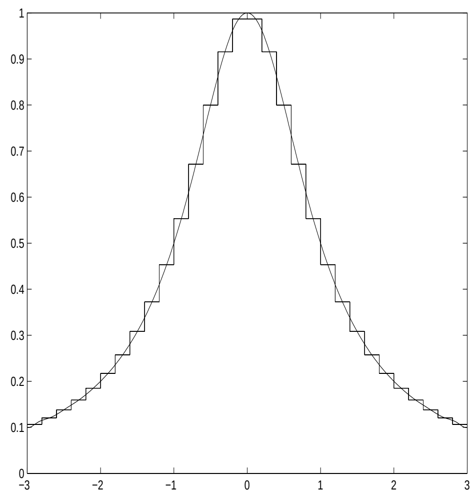

Figure 5: Solution of MVI problem (solid) with prescribed mean values (stairs function)

4 Local and quasilocal splines

4.1 Local quartic splines

A local spline $s_{43}(x) \in C^1$ we can obtain as the result of the interpolation of a given general (independent) data $\{g_i, s_i, s_{i+1}, m_i, m_{i+1}\}$ in FVI, MVI, DVI problems with quartic spline (interpolant has one free parameter at DVI problem) using local spline representation (g, s, m) implemented in M-file **s4gsmval**
.

4.2 Quasilocal quartic splines

Quasilocal splines $s_{42}(s)$ we can obtain (see [9]), when we prescribe some part of local parameters (e.g. function values) and compute the remaining part (e.g. second derivatives) from continuity conditions (for the first derivatives) in knots.

Problem 1:

Let us have given knots $\mathbf{x} = \{x_i,\ i = 0(1)n+1\}$ *and points* $\mathbf{t} = \{t_i,\ i = 0(1)n\}$ *such that* $x_i < t_i < x_{i+1}$ *,* $i = 0(1)n$*, and function values* $\mathbf{s} = \{s_i,\ i = 0(1)n+1\}$ *in* $\mathbf{x}$ *and* $\mathbf{g} = \{g_i,\ i = 0(1)n\}$ *in* $\mathbf{t}$*. Find a quartic spline with defect two with knots* $\mathbf{x}$ *interpolating function values* $\mathbf{s}$ *and* $\mathbf{g}$*.*

The computing of unknown local parameters **m** at this FVI problem is realized in M-file **s42smfv** . For computing spline function values the M-file **s421mval** or yet mentioned **s4gsmval** can be used.

Example: The FVI quasilocal spline (Fig. 6) can be computed using following sequence of function calls:

```
x=[0.1:0.1:2]; t=[0.15:0.1:1.95]; h=diff(x);
s=sin(log(x)).*sin(2*pi*x); g=sin(log(t)).*sin(2*pi*t);
plot(x,s,'x'); hold on; plot(t,g,'o');
for i=1:length(h) d(i)=(t(i)-x(i))/h(i); end;
for i=1:length(h)-1 p(i)=h(i)/h(i+1); end;
[m]=s42smfv(h,d,p,g,s,-7.709,4.015);
xx=[0.1:0.01:2]; yy=zeros(size(xx));
for i=1:length(yy) yy(i)=s4gsmval(1,x,t,s,g,m,xx(i)); end;
plot(xx,yy);
```

Problem 2:

Let us have given knots $\mathbf{x} = \{x_i,\ i = 0(1)n+1\}$ *and function values* $\mathbf{s} = \{s_i,\ i = 0(1)n+1\}$ *in* $\mathbf{x}$ *and mean values* $\mathbf{g} = \{g_i,\ i = 0(1)n\}$*. Find a quartic spline with defect two interpolating function values* $\mathbf{s}$ *and mean values*$\mathbf{g}$*.*

The unknown parameters **m** of this MVI spline can be computed using M-file **s421smmv** which is common for splines with defects $d = 1,\ d = 2$. For computing spline function values the M-file **s421mval** or yet mentioned **s4gsmval** is used.

Example The MVI quasilocal spline (Fig. 7) can be computed using following sequence of function calls:

```
x=[0:pi/8:2*pi]; h=diff(x);
s=sin(x); g=diff(-cos(x))./h; m=zeros(size(s));
plot(x,s,'x'); hold on; stairs(x,[g,g(length(g))]);
[s,m]=s421smmv(h,g,s,m,2);
xx=[0:pi/60:2*pi]; yy=zeros(size(xx));
```

```
for i=1:length(yy) yy(i)=s421mval(xx(i),x,g,s,m); end;
plot(xx,yy);
```

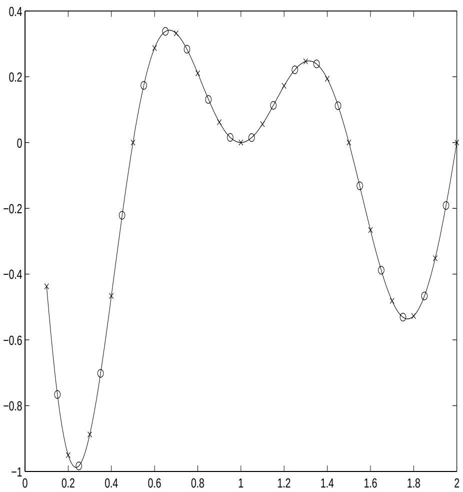

Figure 6: The solution (solid line) of FVI quasilocal problem 1 with different knots (x-mark) and points of interpolation (circle)

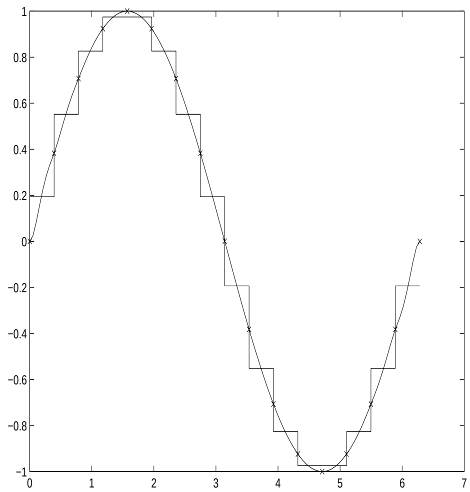

Figure 7: The solution (solid line) of MVI quasilocal problem 2 with prescibed mean values (stairs function)

Problem 3:
Let us have given knots $\mathbf{x} = \{x_i,\ i = 0(1)n+1\}$ *and function values* $\mathbf{s} = \{s_i,\ i = 0(1)n+1\}$ *and values of the first derivatives* $\mathbf{m} = \{m_i,\ i = 0(1)n+1\}$ *in* $\mathbf{x}$*. Find a quartic spline with defect two interpolating function values* $\mathbf{s}$ *and the first derivatives* $\mathbf{g}$*.*

This problem can be solved using (g, s, m) representation of FVI or MVI problems. The algorithms for such problems are realized in M-file **s42qlc**. Any spline function value can be then computed with M-file **s4gsmval**.

Example: The DVI quasilocal spline (Fig. 8) can be computed using following sequence of function call:

```
x=[-2:0.5:5]; s=exp(x).*sin(2*x); m=exp(x).*(sin(2*x)+2*cos(2*x));
h=diff(x);  plot(x,s,'o'); hold on;
[g]=s42qlc(h,0.0523848,s,m,2);
plot(x,s,'o'); hold on;
xx=[-2:0.01:5]; yy=zeros(size(xx));
for i=1:length(xx) yy(i)=s421mval(xx(i),x,g,s,m); end;
plot(xx,yy);
```

Problem 4:
Let us have given knots $\mathbf{x} = \{x_i,\ i = 0(1)n+1\}$ *and function values* s_{2i}*, values*

of the first derivatives m_{2i} and values of the second derivatives M_{2i} in knots x_{2i} and the function values s_{2i+1} in knots x_{2i+1}. Find a quartic spline with defect two interpolating function values s_{2i} and s_{2i+1}, the first derivatives m_{2i} and the second derivatives M_{2i}.

Problem 5:
Let us have given knots $\mathbf{x} = \{x_i,\ i = 0(1)n+1\}$ and function values s_{2i}, values of the first derivatives m_{2i} and values of the second derivatives M_{2i} in knots x_{2i} and values of the first derivatives m_{2i+1} in knots x_{2i+1}. Find a quartic spline with defect two interpolating function values s_{2i}, the first derivatives m_{2i} and and m_{2i+1} and the second derivatives M_{2i}.

Problem 6:
Let us have given knots $\mathbf{x} = \{x_i,\ i = 0(1)n+1\}$ and function values s_{2i}, values of the first derivatives m_{2i} and values of the second derivatives M_{2i} in knots x_{2i}. Find a quartic spline with defect two interpolating function values s_{2i}, the first derivatives m_{2i} and the second derivatives M_{2i}.

The remaining local parameters **s,m** can be computed using M-file **s42loc** in problems 4,5,6. In problem 6 we can even to reach the continuity of the third derivative in knots x_{2i+1}. The details can be found in [8]. For computing spline function values the M-file **fvs42loc** can be used.

Example: The quasilocal spline (Fig. 9) from problem 4 can be computed using following sequence of function call:

```
x=[-5:0.5:5]; s=x.^4-50*x.^2+625; m=4*x.^3-100*x; M=12*x.^2-100;
plot(x,s,'o'); hold on;
plot(x(1:2:length(x)),s(1:2:length(x)),'x');
[s,ms]=s42loc(x,s,m,M,0);
xx=[-5:0.01:5]; yy=zeros(size(xx));
for i=1:length(xx) yy(i)=fvs42loc(xx(i),x,s,ms,M); end;
plot(xx,yy);
```

5 Quartic smoothing spline

The smoothing splines are used for approximation of noisy data. They give some compromise between an interpolation of prescribed values and a least squares approximation of them, regulated by smoothing parameter α (see [1]). The statement of the general smoothing problem is given e.g in [4], [16]. The construction of classical smoothing splines is based on extremal properties of some interpolatory splines (presented in variational theory of splines). For odd degree splines the problem of function values smoothing was stated and solved - for the most

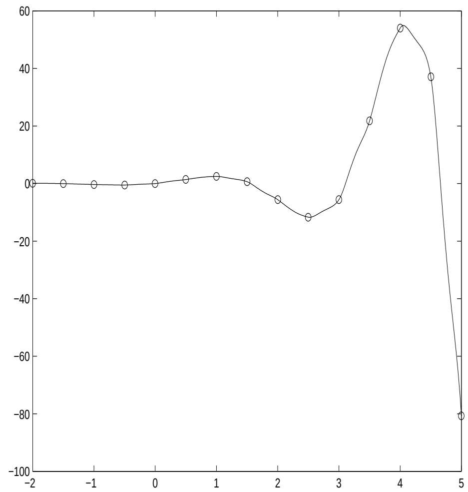

Figure 8: The solution (solid line) of DVI quasilocal problem 3 on simple grid (circle)

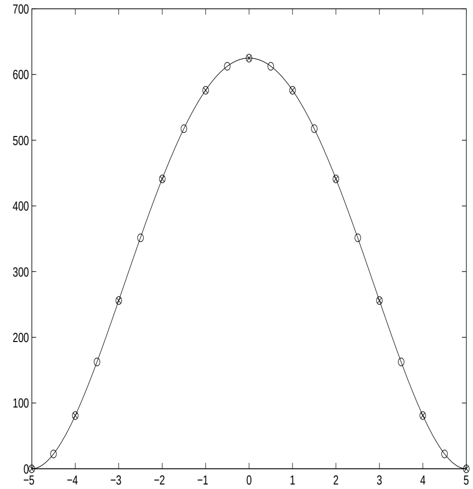

Figure 9: The solution (solid line) of quasilocal problem 4 with knots denoted by circle and point of interpolation by x-mark

known cubic spline smoothing see e.g. [1],[2] or [3]. The problem of mean values smoothing for even degree splines was stated and solved e.g. in [11], [14] for quadratic splines, in [5] and [12] for quartic splines. In this section the algorithm for mean-value smoothing quartic splines on general knot sequence is described (see [12]).

Problem:
Let us have given a knot sequence $\mathbf{x} = \{x_i,\ i = 0(1)n+1\}$ *with prescribed values* $\mathbf{g} = \{g_i,\ i = 0(1)n\}$ *positive weight coefficients* $\mathbf{w} = \{w_i,\ i = 0(1)n+1\}$ *and some smoothing parameter* $\alpha > 0$. *Find a quartic spline* s_{41} *smoothing the mean values* $\mathbf{g}$ *i.e*

$$\textit{minimize} \quad \int_{x_0}^{x_{n+1}} [s''(x)]^2 dx + \alpha \sum_{i=0}^{n} w_i (g_i - p_i)^2$$

where p_i *are unknown mean values of spline* $s(x)$.

The computing local parameters of quartic smoothing splines under previous algorithms is implemented in function **spl4vs**. The spline function values can be then computed with M-file **spl4hodn** .

Example: The MVS complete spline (Fig. 10) with $\alpha = 0.01$ can be computed using following sequence of function calls:

```
alpha=0.001;
[m,M,p]=spl4vs(2,[1,-1;1,1],alpha,x,gd);
```

```
xx=[-5:0.01:5]; yy=zeros(size(xx));
for i=1:length(xx) yy(i)=spl4hodn(xx(i),m,M,x,p); end;
plot(xx,yy,'g:');
```

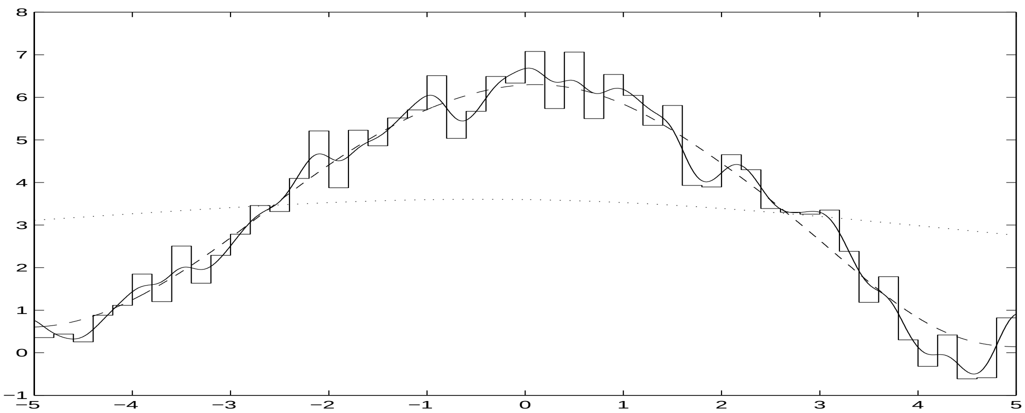

Figure 10: The solution of MVS problem for $\alpha = 0.01$ (dotted), $\alpha = 1$ (dashed), $\alpha = 1000$ (solid)

6 Smoothing under conditions of convexity and monotonicity

In this section the problem of shape preserving smoothing mean values by quartic splines is shown using (g, m, M) representation (for more detailed description see [17]). Under smoothing we understand here that resulting splines minimize some smoothing functionals only on some subset of quartic splines space.

Problem 1:
Let us have given a knot sequence $\mathbf{x} = \{x_i,\ i = 0(1)n+1\}$ *with prescribed values* $\mathbf{g} = \{g_i,\ i = 0(1)n\}$ *positive weight coefficients* $\mathbf{w} = \{w_i,\ i = 0(1)n+1\}$ *and some smoothing parameter* $\alpha > 0$. *Find a increasing quartic spline* s_{41} *or* s_{42} *smoothing the mean values* $\mathbf{g}$.

Problem 2:
Let us have given a knot sequence $\mathbf{x} = \{x_i,\ i = 0(1)n+1\}$ *with prescribed values* $\mathbf{g} = \{g_i,\ i = 0(1)n\}$ *positive weight coefficients* $\mathbf{w} = \{w_i,\ i = 0(1)n+1\}$ *and some smoothing parameter* $\alpha > 0$. *Find a convex quartic spline* s_{41} *or* s_{42} *smoothing the mean values* $\mathbf{g}$.

These problems are solved by function **s4shapel** which uses the function **qp** from Matlab's Optimization Toolbox. The spline function value in any point can be then computed with M-file **spl4hodn**.

Example: The increasing MVS spline (Fig. 11) with $\alpha = 0.01$ can be computed using following sequence of function calls:

```
functional='second derivatives'; shape='increase '; alfa=0.1;
[m,M,p,J1]=s4shapel(1,shape,functional,alfa,x,g);
hxx=[x(1):pom:x(length(x))]; hyy=zeros(size(hxx));
for i=1:length(hxx)  hyy(i)=spl4hodn(hxx(i),m,M,x,p); end;
plot(hxx,hyy,'g:');
```

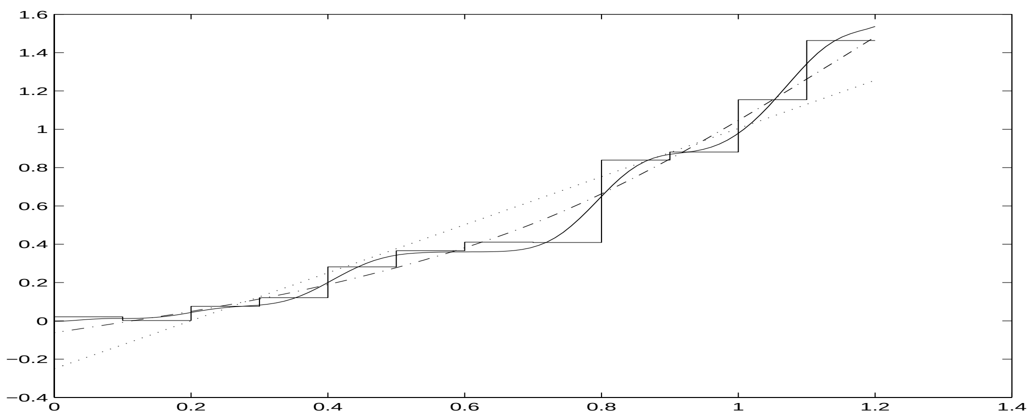

Figure 11: Increasing MVS quartic spline for $\alpha = 0.1$ (dotted), $\alpha = 100$ (dashed), $\alpha = 1e6$ (solid)

7 M-files overwiev for quartic splines

For some more detailed description of these M-files see [13].

Function	*computed values*
s4gval	FV s(t) on simple grid (from LP t,x,s,m,M)
s41dmM	LP m, M of FVI quartic spline on simple grid
lp1ds4e	LP of FVI quartic spline on simple equidistant grid
s42deval	FV s(x,y) in 2D from 25 local parameters
lps42d	2D local parameters on rectangular equidist. simple knotset

s4gsmval	FV of s(x) with LP g,s,m - for FVI, MVI, DVI problems
s41smfvi	LP s,m from data g + BC (t,x - FVI)
spl4hodn	FV s(x) from LP g,m,M (FVI, MVI)
spl4hodd	FV s(x) from LP g,s,M (DVI)
spl4	LP m,M from data g + BC (FVI)
spl4d	LP s,M from data g+BC (DVI)
spl4m	LP m,M from data g+BC (MVI)
s4mvise	LP s,m from data g+BC (MVI, MVS)
s42smfv	LP m from data g,s of s42 (FVI)
s421smmv	LP (s),m of s42 (s41) - (FVI)
s421mval	FV of s42 (s41) , (FVI)
s42qlc	LP g of quasilocal spline from data s,m - (FVI, MVI)
s42loc	LP of local s42
fvs42loc	FV of local s42
spl4vs	LP of MVS spline
s4shapel	LP of shape preserving MVS spline

LP - local parameters
s,m,M - function values, first and second derivatives in spline knots
FV, MV, DV - function, mean, derivative values
FVI, MVI, DVI - interpolation of spline function , mean, derivative values
BC - boundary conditions

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Author's address:

Jiří Kobza,
Dept. MA&AM , Faculty of Sciences, UP Olomouc,
Tomkova 40, 779 00 Olomouc, Czech Republic
E-mail: Kobza@risc.upol.cz

Pavel Ženčák
Dept. MA&AM , Faculty of Sciences, UP Olomouc,
Tomkova 40, 779 00 Olomouc, Czech Republic
E-mail: zencak@matnw.upol.cz