

# MULTIRESOLUTION TIME-DOMAIN ANALYSIS OF MICROWAVE STRUCTURES USING MATLAB

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## Abstract

*In this paper, numerical analysis of microwave structures using multiresolution time-domain (MRTD) method in MATLAB environment is presented. Commonly used scaling and wavelet orthogonal systems are introduced. Corresponding algorithm and its implementation in the form of m-file are described. Finally, accuracy and computational demands of the proposed program are discussed.*

## Introduction

Nowadays, the most exploited method for time-domain numerical analysis of microwave structures is the well-known finite-difference time-domain (FDTD) method [1]. It is accurate enough for a wide variety of problems and at the same time straightforward to implement, which was successfully demonstrated in [2]. This technique has, however, certain limitations in describing fast variations of the electromagnetic field in the computational domain. In such cases, sufficient degree of accuracy is reached only by utilizing very fine meshes, which leads to high computer memory demands.

The multiresolution time-domain (MRTD) method uses expansion of field distribution into scaling functions and wavelet functions of the same and higher resolution levels [3]. Using the wavelet basis enables us to properly describe the areas where the field changes rapidly. Nevertheless, the computer burden does not increase significantly, as most of the wavelet coefficients are small enough to be neglected. The implementation of the MRTD basic algorithm in MATLAB environment is object of this paper.

## Theory

The electromagnetic field in the computational domain, represented by  $f(x)$ , can be approximated by generally infinite series of scaling and wavelet functions,

$$f(x) = \sum_{n=-\infty}^{\infty} c_n \phi_{m_0 n}(x) + \sum_{m=m_0+1}^{\infty} \sum_{n=-\infty}^{\infty} d_{mn} \psi_{mn}(x), \quad (1)$$

where  $\phi$  and  $\psi$  represent the scaling and wavelet functions respectively, with  $n$  as a coordinate of translation and  $m$  as wavelet resolution level ( $m_0$  is the primary resolution level). From (1) it can be seen that the appropriate choice of the scaling and wavelet functions affects the precision of field approximation.

The typical scaling and wavelet orthogonal systems are described below.

- **Haar system** – scaling function in space or time domain is a pulse function, while the Fourier transform is the well-known *sinc* function. This basis is localized in space or time but not in the spectrum, which yields negative effects at discretization. Using Haar scaling functions only we obtain the FDTD method, which is apparently a special case of MRTD.
- **Shannon system** is the opposite of Haar system, as its spectrum is formed by a pulse function and the original domain expansion is done by the *sinc*. Shannon basis is perfectly smooth but not localized in space or time.

- **Battle-Lemarie system** is a compromise between the two preceding systems: it is fairly localized in both space/time and frequency domains.

In the following, an algorithm based on Battle-Lemarie orthogonal system will be described. Only the scaling functions were incorporated for simplicity. Further theoretical background of the MRTD method can be found in [3].

## Formulation

The formulation of MRTD is based on the first two Maxwell equations, where the derivatives are expressed from the expansion (1). The Battle-Lemarie system was used for expansion in space domain, while pulse functions were utilized in time domain, which saves the amount of storage needed. For example, the two-dimensional case with TM mode is considered. The multiresolution time-domain equation system for lossless media can then be written as [3]

$$H_x|_{i,j-1/2}^{n+1/2} = H_x|_{i,j-1/2}^{n-1/2} - \frac{\Delta t}{\mu \Delta} \sum_{j'=j-n_a}^{j+n_a-1} \alpha(j') E_z|_{i,j'}^n, \quad (2a)$$

$$H_y|_{i-1/2,j}^{n+1/2} = H_y|_{i-1/2,j}^{n-1/2} + \frac{\Delta t}{\mu \Delta} \sum_{i'=i-n_a}^{i+n_a-1} \alpha(i') E_z|_{i',j}^n, \quad (2b)$$

$$E_z|_{i,j}^{n+1} = E_z|_{i,j}^n + \frac{\Delta t}{\varepsilon \Delta} \left( \sum_{i'=i-n_a}^{i+n_a-1} \alpha(i') H_y|_{i'-1/2,j}^{n+1/2} - \sum_{j'=j-n_a}^{j+n_a-1} \alpha(j') H_x|_{i,j'-1/2}^{n+1/2} \right). \quad (2c)$$

Here,  $H_x$ ,  $H_y$  and  $E_z$  are the involved magnetic and electric field components,  $\Delta$  and  $\Delta t$  are, respectively, the space and time steps and  $\mu$  with  $\varepsilon$  are the material parameters (permeability and permittivity). Indices  $i, j$  denote the space coordinates and  $n$  denotes the time coordinate. Finally, the  $\alpha$  coefficients relate to the scaling functions expansion with  $n_a$  as a maximum of the terms incorporated in the algorithm (ideally infinite number).

## Application

The described algorithm was implemented in the form of MATLAB function. The program was highly vectorized in order to achieve the best performance in MATLAB.

The program kernel for 2D TM mode (2) is written as follows:

```

% matrix indices preprocessing
ind = 1:(xmax*ymax);
ind = reshape(ind,xmax,ymax);

...

for t = 1:Nt, % time-marching cycle

    % indices initialization
    ind1 = [ind(1:xmax-1,:) ; -ind(xmax-1,:)];
    ind2 = [ind(1,:) ; -ind(1:xmax-1,:)];
    ind3 = [ind(:,1:ymax-1) -ind(:,ymax-1)];
    ind4 = [ind(:,1) -ind(:,1:ymax-1)];

    for i = 1:na, % alpha cycle

        Ez = Ez + C1(i).*( Hy(abs(ind1)) - Hy(abs(ind2)) ...
            - Hx(abs(ind3)) + Hx(abs(ind4)) );
    end
end

```

```

ind1 = ind1 + (ind1>0);
mask = ismember(abs(ind1),PEC);
ind1(mask) = -ind1(mask);
ind1 = ind1 + (ind1<0);

ind2 = ind2 + (ind2>0);
mask = ismember(abs(ind2),PEC);
ind2(mask) = -ind2(mask);
ind2 = ind2 + (ind2<0);

ind3 = ind3 + xmax.*(ind3>0);
mask = ismember(abs(ind3),PEC);
ind3(mask) = -ind3(mask);
ind3 = ind3 + xmax.*(ind3<0);

ind4 = ind4 + xmax.*(ind4>0);
mask = ismember(abs(ind4),PEC);
ind4(mask) = -ind4(mask);
ind4 = ind4 + xmax.*(ind4<0);

end;

Ez(PEC) = 0;           % PEC boundary condition

...

% indices initialization
ind1 = [ind(2:xmax-1,:) ; -ind([xmax xmax],:)]';
ind2 = [ind(1,:) ; -ind(2:xmax,:)]';
ind3 = [ind(:,2:ymax-1)   -ind(:,[ymax ymax])]';
ind4 = [ind(:,1)   -ind(:,2:ymax)]';

for i = 1:na;           % alpha cycle

Hy = Hy + C2(i).*( sign(ind1).*Ez(abs(ind1)) ...
                   + sign(ind2).*Ez(abs(ind2)) );
Hx = Hx - C2(i).*( sign(ind3).*Ez(abs(ind3)) ...
                   + sign(ind4).*Ez(abs(ind4)) );

ind1 = ind1 + 1;
mask = ismember(abs(ind1),PEC);
ind1(mask) = -ind1(mask);

ind2 = ind2 + 1;
mask = ismember(abs(ind2),PEC);
ind2(mask) = -ind2(mask);

ind3 = ind3 + xmax;
mask = ismember(abs(ind3),PEC);
ind3(mask) = -ind3(mask);

ind4 = ind4 + xmax;
mask = ismember(abs(ind4),PEC);
ind4(mask) = -ind4(mask);

end;

end;

```

Here, the  $C1$  and  $C2$  coefficients correspond to the alpha coefficients combined with time and space steps and material properties (2), which minimizes the amount of arithmetical operations. The indices change with the coefficients to properly model the boundary condition made of perfect electric conductor (layout included in `PEC`), according to the image principle. Maximum memory savings are achieved by changing the indices in cycle and using their sign to indicate the sense of incrementation and polarity of image. Even more storage can be reserved by using less extensive data types such as `int16`, although the program is then fairly complicated.

### Results

The constructed 2D code was used to analyze the wavemode frequencies of microwave transmission line, particularly the shielded air-strip line analyzed in [2]. Cross section of the structure is displayed in Fig. 1.

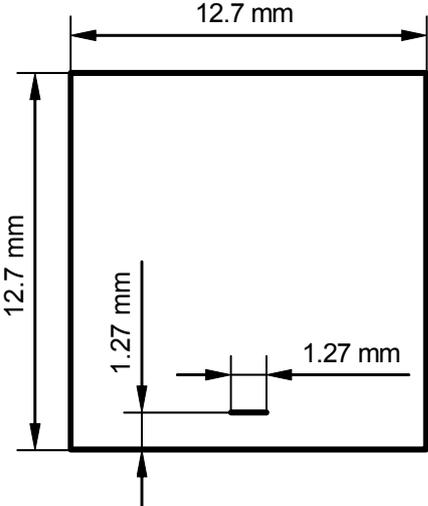


Fig. 1: Cross section of the analyzed shielded air-strip line

The corresponding magnitude spectrum obtained through the Fourier transform (particularly by the FFT) is shown in Fig. 2. The area of transverse sections was discretized with 20 cells per side. The problem was solved in frequency range from 0 to 50 GHz, with 10 MHz resolution.

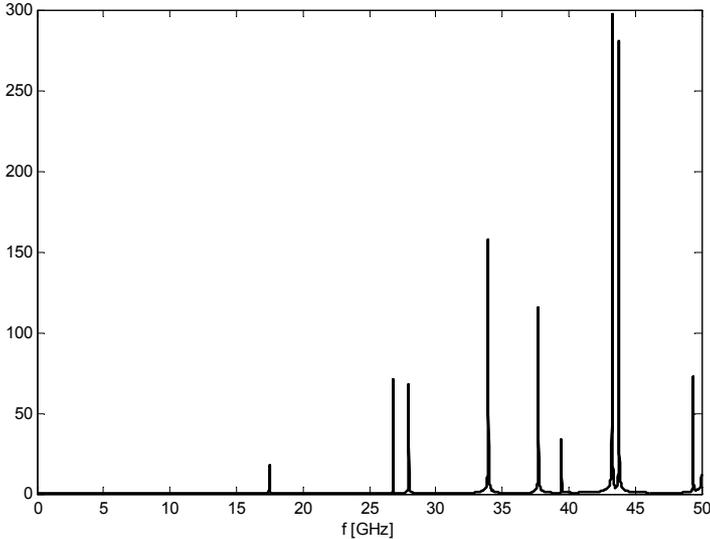


Fig. 2: Frequency spectrum for shielded air-strip line, 20 cells per side

The obtained results appeared to be more accurate than with the original FDTD method used for comparison, but, on the other hand, the running time was sacrificed. The total computation time was 22 min., whereas the second order FDTD method managed to compute the same structure within 32 sec, even with higher frequency resolution and on slower machine.

## **Conclusion**

This paper follows the work [2] in proving that even the multiresolution time-domain method can be implemented in MATLAB with advantage. This fact has been demonstrated on the two-dimensional TM mode case, the TE mode or the full 3D case can be obtained similarly, however. The algorithm can also be slightly modified to accommodate the wavelet functions and, in this way, to improve the accuracy where needed.

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