DYADIC WEIGHT NEURAL NETWORK FOR 2D IMAGE PROCESSING

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Abstract. *The paper is devoted to fuzzy image processing based on Łukasiewicz algebra with square root. A method of image processing based on dyadic weight neural network with fuzzy logic function preprocessing in a hidden layer is presented. Results are demonstrated on 2D biomedical images. All the fuzzy algorithms are realized in the MATLAB environment.*

Keywords: *nonlinear filter, neural network, fuzzy network, 2D image, MRI, de-noising, Matlab.*

1 Preliminaries

The problem of 2D biomedical images de-noising arises if the low-resolution apparatus is used. The noise reduction and signal structure saving are contradictory but useful aims. Fuzzy systems are able to produce output near to this aim. The Łukasiewicz algebra with square root (ŁA_{sqrt}) was chosen as mathematic model for fuzzy system realization.

Łukasiewicz algebra [2] is an MV-algebra operating on $[0, 1]$ interval using conjunction, disjunction, multiplication and residuum as basic logic operators. This MV-algebra was enriched by a square root function because of low sensitivity system construction and weighted compromise making:

$$\text{ŁA}_{\text{sqrt}} = \langle \mathbf{L}, \wedge, \vee, \otimes, \rightarrow, \text{sqrt}, 0, 1 \rangle$$

where $\mathbf{L} = [0, 1]$. Basic logic operators and functions are summarized in a Tab. 1. A small number of basic operators brings to easy hardware realization of fuzzy systems.

It is useful to define derived operators in ŁA_{sqrt} for simple notation of fuzzy expressions. The list of derived operators is shown in a Tab. 2.

operator/function	**name**	**definition**
$\wedge$	conjunction	$a \wedge b = \min(a, b)$
$\vee$	disjunction	$a \vee b = \max(a, b)$
$\otimes$	Łukasiewicz multiplication	$a \otimes b = \max(a + b - 1, 0)$
$\rightarrow$	residuum	$a \rightarrow b = \min(1 - a + b, 1)$
sqrt	square root function	$\text{sqrt}(a) = (1 + a)/2$

Table 1: Basic operators and functions in ŁA_{sqrt} $(a, b \in \mathbf{L})$

A *fuzzy logic function* (FLF) in ŁA_{sqrt} is composed from constants and free variables from $\mathbf{L}$ and finite number of basic ŁA_{sqrt} operators and functions. A *sensitivity* of FLF $\varphi : \mathbf{L}^n \rightarrow \mathbf{L}$ is defined as

$$\lambda = \max_{\boldsymbol{x} \neq \boldsymbol{y}} \frac{\varphi(\boldsymbol{x}) \circ \varphi(\boldsymbol{y})}{\sum\limits_{k=1}^{n} |x_k - y_k|} \quad \text{where } \boldsymbol{x}, \boldsymbol{y} \in \mathbf{L}^n.$$

operator	name	definition
$\neg$	negation	$\neg a = a \to 0$
$\leftrightarrow$	equivalence (biresiduum)	$a \leftrightarrow b = (a \to b) \wedge (b \to a)$
$\circ$	non-equivalence (distance)	$a \circ b = \neg(a \leftrightarrow b)$
$\oplus$	addition	$a \oplus b = \neg(\neg a \otimes \neg b)$
$\ominus$	subtraction	$a \ominus b = a \otimes \neg b$
$\odot$	multiplication by integer	$n \odot a = \underbrace{a \oplus a \oplus \cdots \oplus a}_{n},\ 0 \odot a = 0$
a^n	integer power	$a^n = \underbrace{a \otimes a \otimes \cdots \otimes a}_{n},\ a^0 = 1$

Table 2: Derived operators in ŁA_{sqrt} $(a, b \in \mathbf{L}, n \in \mathbf{N})$

Any FLF is Lipschitz continuous function as proven in [3]. This property allows to construct systems with low sensitivity to input data. Thus, the ŁA_{sqrt} seems to be an efficient tool for the 2D image de-noising.

2 Dyadic Weight Neural Network

The image processing using the FLFs can be perceived as a hierarchical process without loops. Its representation by oriented acyclic graphs is recommended. A resulting FLF structure can be called *fuzzy logic function network*. It consists of independent layers with interconnections. The signals from pixel neighborhood come into the first input layer. The enhanced pixel signal is produced by output node in output layer. It is necessary to use one hidden layer at least for advanced signal processing. One of three-layered networks is based on a compromise in ŁA_{sqrt} using weighted average with fixed dyadic weights:

DEFINITION 1. *Let $n, H \in \mathbf{N}$ be numbers of input and hidden nodes, $\boldsymbol{x} = (x_1, \dots, x_n) \in \mathbf{L}^\mathbf{n}$, $\boldsymbol{f} = (f_1, \dots, f_H) \in \mathbf{L}^H$. Let $f_i = \mathrm{f}_i(\boldsymbol{x})$ where f_i be FLF for $i = 1, \dots, H$. Let $y = \sum_{k=1}^{H} w_k \mathrm{f}_k(\boldsymbol{x})$ be an output signal. Let $w_k = m_k/2^N$ where $m_k, N \in \mathbf{N}_0$ and $\sum_{k=1}^{H} w_k = 1$. Then the structure producing y from $\boldsymbol{x}$ by $\boldsymbol{f}$ is called **dyadic weight neural network** (DWNN).*

The structure of DWNN is depicted in the Fig. 1.

2.1 Weights Optimization

The DWNN weight vector $\boldsymbol{w}$ can be subject of discrete optimization for fixed exponent N. A good initial estimate of weights can be obtained using linear regression in three steps:

(i) non-dyadic weights are obtained by the minimization of the sum of squares:

$$\mathrm{ssq}(w_1, \dots, w_H) = \min$$

$$\sum_{k=1}^{H} w_k = 1$$

$$w_k \geq 0 \text{ for } k = 1, \dots, H$$

(ii) rounding of weights to the nearest dyadic values

(iii) final dyadic normalization

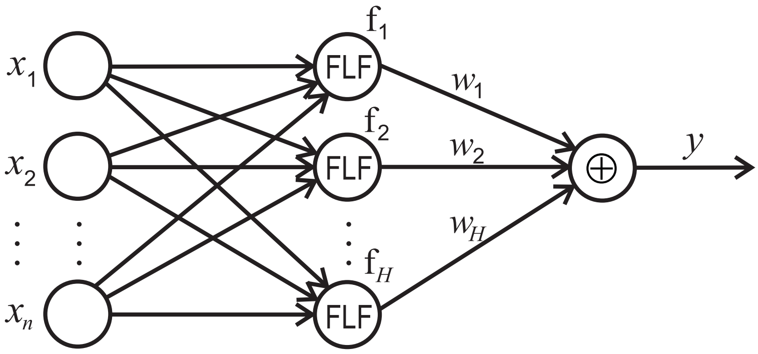

Figure 1: Structure of DWNN

2.2 DWNN Properties

LEMMA 1. *Let $n \in \mathbf{N}$. Let $\boldsymbol{x} \in \mathbf{L}^n$. Let $N, m_k \in \mathbf{N}_0$ for $k = 1, \ldots, n$. Let $w_k = m_k/2^N$ be dyadic weights for $k = 1, \ldots, n$ and $\sum_{k=1}^{n} w_k \leq 1$. Then any function*

$$\mathrm{f}(\boldsymbol{x}) = \sum_{k=1}^{n} w_k x_k$$

is a FLF.

Proof. Let $n \in \mathbf{N}$ and $N \in \mathbf{N}_0$. Let $\boldsymbol{x} \in \mathbf{L}^n$ and $\boldsymbol{m} \in \mathbf{N}_0^n$. Let $\boldsymbol{w} = \boldsymbol{m}/2^N$ and $\sum_{k=1}^{n} w_k \leq 1$. Then

$$w_k \cdot x_k = \frac{m_k}{2^N} \cdot x_k = m_k \odot \frac{x_k}{2^N}$$

and

$$\sum_{k=1}^{n} w_k \cdot x_k \leq \sum_{k=1}^{n} w_k \leq 1.$$

Then the traditional summation can be substituted by the $\oplus$ operator and

$$\mathrm{f}(\boldsymbol{x}) = \sum_{k=1}^{n} w_k \cdot x_k = \bigoplus_{k=1}^{n} \left(m_k \odot \frac{x_k}{2^N} \right)$$

is FLF. □

THEOREM 2. *The output of DWNN is FLF of its input $\boldsymbol{x}$.*

Proof. Let $\mathrm{y} : \mathbf{L}^n \to \mathbf{L}$ and $y = \sum_{k=1}^{H} w_k \mathrm{f}_k(\boldsymbol{x})$ be a DWNN output. Let $w_k = m_k/2^N$ where $m_k, N \in \mathbf{N}_0$ and $\sum_{k=1}^{H} w_k = 1$.

Any f_k is FLF of $\boldsymbol{x}$ and y is a weighted average of $f_1, \ldots, f_H$ with positive dyadic weights. According to lemma 1 and FLF definition, the function y is FLF. □

THEOREM 3. *Let $H \in \mathbf{N}$ and λ_k be a sensitivity of f_k for $k = 1, \ldots, H$. Then the sensitivity of DWNN is*

$$\lambda \leq \max_{1 \leq k \leq H} (\lambda_k).$$

Proof. See [3]. □

3 DWNN Testing on 2D Biomedical Images

The DWNN was tested on artificial and real images. The artificial biomedical image (Fig. 3) was obtained using addition of noise from top left corner of real MRI image. The quality of filtering was computed using a signal to noise ratio (SNR) criterion[1]. Results are collected in a Tab. 4.

The hidden layer of DWNN contains 5 individual FLF filters as described in a Tab. 3. The weights of DWNN were subject of optimization for biomedical image with SNR criterion. The resulting weight vector is $w = (0, 17/32, 15/32, 0, 0)$, i. e. only two filters were used in DWNN. The quality of DWNN filtering is also included in the Tab. 4. The de-noising with DWNN is better than the de-noising with individual filters. The last row in the Tab. 4 allows to compare filtering quality with traditional FIR filter (binomial FIR filter). The Figs. 3–7 demonstrate the biomedical image before and after de-noising.

Filter	**FLF**	**mask**[2]
F_1	median	1-1-1
F_2	quasi median[3] ($k = 1$)	1-1-1
F_3	quasi median[3] ($k = 1$)	4-2-1
F_4	quasi median[3] ($k = 2$)	4-2-1
F_5	BES estimation[4]	4-2-1

Table 3: Final set of FLFs

Filter	**SNR [dB]**
NO	10.8381
F_1	14.6766
F_2	15.2065
F_3	15.2262
F_4	14.9529
F_5	14.7934
DWNN	**15.5375**
binomial FIR	14.7205

Table 4: Quality of MRI de-noising

4 Conclusion

The $ŁA_{\text{sqrt}}$ seems to be a good model for fuzzy image de-noising. The DWNN were described as method how to construct weighted average in $ŁA_{\text{sqrt}}$. The filters based on FLF are used individually and then as hidden nodes in DWNN. The quality was computed using SNR criterion. The results demonstrate that optimized DWNN produces output image better than the individual filters.

[1] $\text{SNR} = 10 \cdot \log \frac{\text{Var}(X)}{\text{Var}(Y-X)}$ (X is ideal image and Y is de-noised one).

[2] Square mask of size 3×3 is denoted by weight of central pixel, weight of its neighbors and weight of corners.

[3] Quasi median is computed by $\text{Q}_k(\boldsymbol{x}) = \frac{1}{2} \cdot \left(x_{(\lceil \frac{n+1}{2} \rceil + k)} + x_{(\lfloor \frac{n+1}{2} \rfloor - k)} \right)$.

[3] BES estimation is computed by $\text{BES}(\boldsymbol{x}) = \frac{1}{4} \cdot \left(x_{(\lceil \frac{n}{4} \rceil)} + x_{(\lfloor \frac{n+1}{2} \rfloor)} + x_{(\lceil \frac{n+1}{2} \rceil)} + x_{(\lfloor \frac{3n+4}{4} \rfloor)} \right)$.

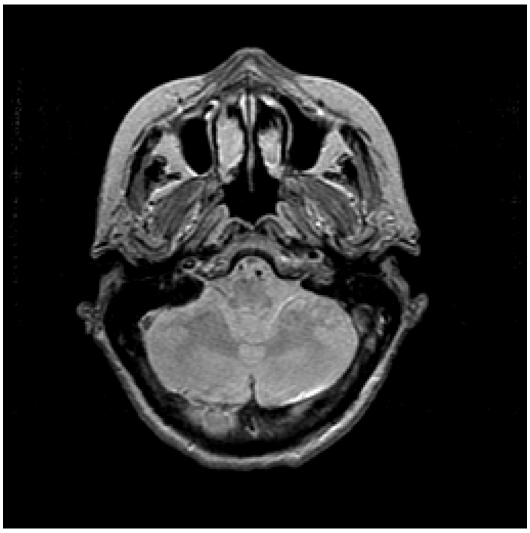

Figure 2: Ideal biomedical image

Figure 3: Noised biomedical image

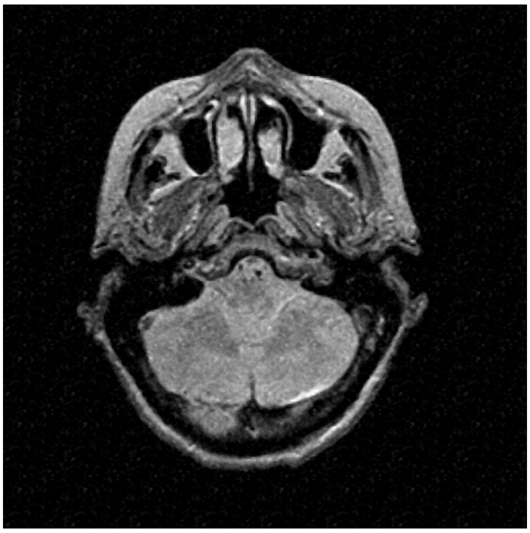

Figure 4: MRI filtered by F_2

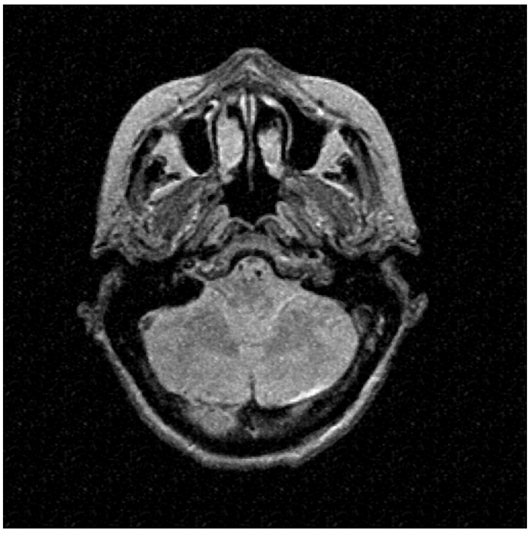

Figure 5: MRI filtered by F_3

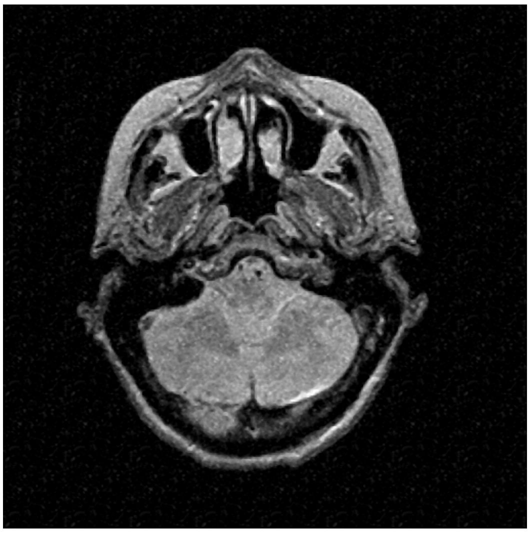

Figure 6: MRI filtered by DWNN

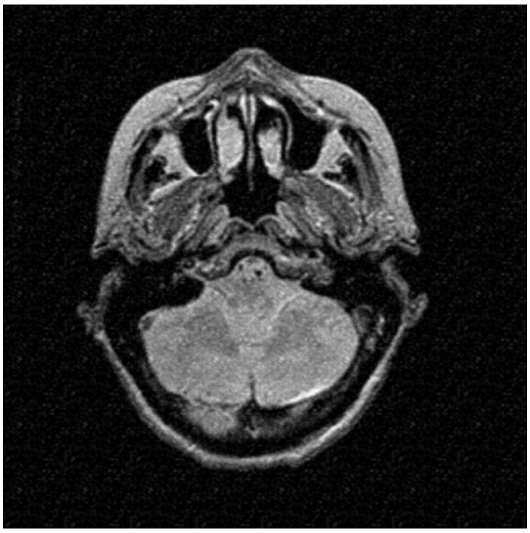

Figure 7: MRI filtered by binomial FIR

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