ROBUST CONTROLLER DESIGN USING EDGE THEOREM FOR MODULAR SERVO SYSTEM

M. Hypiusová, J. Osuský and S. Kajan

Institute of Control and Industrial Informatics, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava, Slovak Republic

Abstract

The paper deals with the design of robust controllers for uncertain SISO systems in the frequency domain. The approach is accomplished with the Edge Theorem and the Neymark D-partition method for the affine model. The proposed method guarantees the required degree of stability. The practical application is illustrated by the robust controller design for a Modular Servo System.

1 Introduction

For many real processes a controller design has to cope with the effect of uncertainties, which very often cause a poor performance or even instability of closed-loop systems. The reason for that is a perpetual time change of parameters (due to aging, influence of environment, working point changes *etc.*), as well as unmodelled dynamics. The former uncertainty type is denoted as the parametric uncertainty and the latter one the dynamic uncertainty. A controller ensuring closed-loop stability under both of these uncertainty types is called a robust controller. A lot of robust controller design methods are known from the literature [1], [2] in the time- as well as in the frequency domains.

The focus of this paper is to show robust PID controller design to control angular velocity of a Modular Servo System for three working points, where two working points are identified with inertia load and one without inertia load. The method is based on the Edge Theorem and the Neymark D-partition method considering uncertain system model with parametric uncertainties. The designer can specify a required closed-loop stability degree.

2 Modular Servo System

The Modular Servo System (MSS) consists of the Inteco digital servomechanism and open-architecture software environment for real-time control experiments [4]. The measurement system is based on the RTDAC4/USB acquisition board equipped with a D/A and A/D converters. I/O board communicates with the power interface unit. The whole logic necessary to activate and read the encoder signals and to generate the appropriate sequence of the PWM pulses to control the DC motor is configured in the Xilinx® chip of the RT-DAC/USB board. All functions of the board are accessed from the Modular Servo Toolbox, which operates directly in the MATLAB Simulink environment [3].

The MSS consists of the following modules arranged in the chain: the DC motor with the generator, inertia load, encoder, magnetic brake and the gearbox with the output disk depicted in Fig. 1. The system has no got an inner feedback for dead zone compensation. The accuracy of the measured velocity is 5% while the accuracy of the angle is 0.1%. The armature voltage of the DC motor is controlled by a PWM signal v(t) excited by a dimensionless control signal in the form $u(t) = v(t)/v_{max}$.

In our experiment backlash module was not applied. The servomechanism is connected to a computer where a control algorithm is realised based on measurements of the angle and angular velocity. In our paper only the angular velocity was controlled.

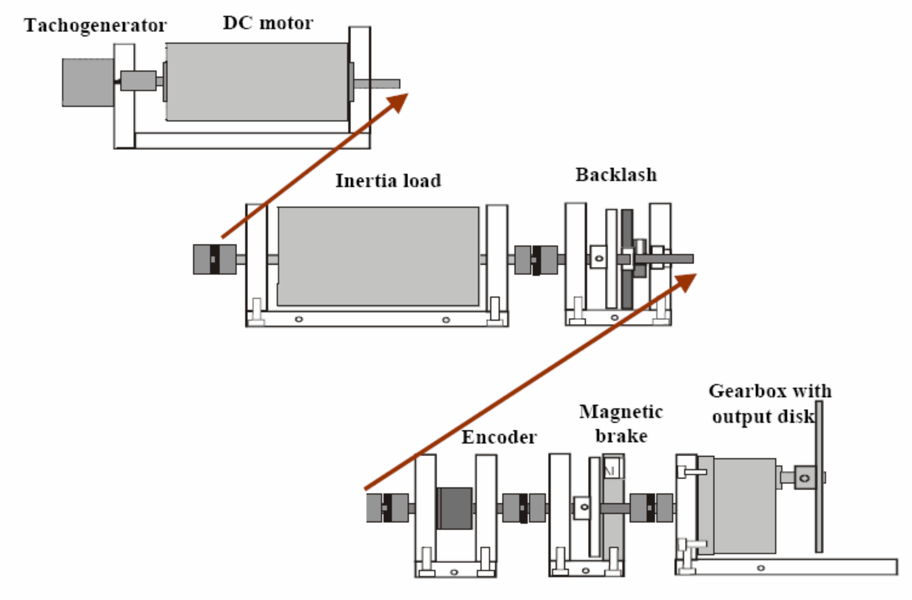

Figure 1: MSS mechanical construction

3 Robust controller design using the Edge Theorem

If a part of coefficients of the plant vary dependently, then it is better to use the affine model of the plant in the form:

$$G(s)=\frac{B(s)}{A(s)}=\frac{b_0(s)+\sum_{i=1}^{p} b_i(s)q_i}{a_0(s)+\sum_{i=1}^{p} a_i(s)q_i} \tag{1}$$

where $q_i \in \left\langle \underline{q}_i, \overline{q}_i \right\rangle$ are uncertain coefficients. The coefficients depend linearly on uncertain parameter vector $\mathbf{q}^T=\left[q_1,\ldots,q_p\right]$. The parameters q_i vary within a p - dimensional box

$$\mathbf{Q}=\left\{\mathbf{q}: q_i \in \left\langle \underline{q}_i, \overline{q}_i \right\rangle, i=1,\ldots,p\right\}. \tag{2}$$

If we vary parameters $q_i=\underline{q}_i$ or $q_i=\overline{q}_i$ then is possible to obtain 2^p transfer functions with constant coefficients; inserting them to the vertices of a p - dimensional polytope, the transfer function (1) describes a so-called *polytopic system*. Consider the controller transfer function in the form

$$G_R(s)=\frac{F_1(s)}{F_2(s)} \tag{3}$$

where $F_1(s)$ and $F_2(s)$ are polynomials with constant coefficients. Then the characteristic polynomials with the polytopic system are

$$p(s,\mathbf{q})=b_0(s)F_1(s)+a_0(s)F_2(s)+\sum_{i=1}^{p} q_i\left[b_i(s)F_1(s)+a_i(s)F_2(s)\right] \tag{4}$$

or in a more general form

$$p(s,\mathbf{q})=p_0(s)+\sum_{i=1}^{p} q_i p_i(s) \qquad q_i \in \mathbf{Q} \tag{5}$$

Theorem 1(Edge Theorem)

The polynomial family (5) is stable if and only if the edges of **Q** are stable. □

The simple stability analysis method for families of polynomials (edges of **Q**) is given in the following theorem.

Theorem 2(Bialas)

Let $H_n^{(a)}$ and $H_n^{(b)}$ be the Hurwitz matrices of

$$\begin{aligned} p_b(s) &= p_{b0} + p_{b1}s + p_{b2}s^2 + \ldots + p_{bn}s^n \quad p_{bn} > 0, \\ p_a(s) &= p_{a0} + p_{a1}s + p_{a2}s^2 + \ldots + p_{an}s^n \quad p_{an} > 0, \end{aligned} \tag{6}$$

respectively. The polynomial family

$$p(s,Q) = \{\lambda p_a(s) + (1-\lambda)p_b(s), \quad \lambda \in [0,1]\} \tag{7}$$

is stable if and only if:

1. $p_b(s)$ is stable
2. the matrix $\left(H_n^{(b)}\right)^{-1} H_n^{(a)}$ has no nonpositive real eigenvalues. □

Using the Edge Theorem, the controller design has to be applied to 4 vertices of the polytopic system; by applying e.g. the Neymark's D-partition method guaranteeing the required closed-loop stability degree we choose the controller coefficients such that the vertices of polytopic system are stable. Then we have to check stability of each edge of the box **Q** by e.g. the Bialas Theorem. If any of the edges is unstable, new controller coefficients are to be chosen by Neymark's method.

4 Design of robust controller for Modular Servo System

Consider the transfer functions of a angular velocity of the Modular Servo System obtained by identification in three working points:

WP1: with inertia load;

manipulated variable $u = 0.4$ [V]; regulated variable $y = 54$ [rad/s];

$$G_{p_1}(s) = \frac{-167.7s + 25730}{s^2 + 77.69s + 124.8}$$

WP2: with inertia load $u = 0.7$ [V]; $y = 117.5$ [rad/s];

$$G_{p_2}(s) = \frac{-52.7s + 10690}{s^2 + 38.15s + 44.79}$$

WP3: without inertia load $u = 0.7$ [V]; $y = 145$ [rad/s];

$$G_{p_3}(s) = \frac{-7444s + 2528000}{s^2 + 194.8s + 11240}$$

The Edge Theorem based approach uses the polytopic model:

$$G_p(s) = \frac{b_0(s) + b_1(s)q_1 + b_2(s)q_2}{a_0(s) + a_1(s)q_1 + a_2(s)q_2}$$

where: $b_0(s) = -3748.35s + 1269345$, $a_0(s) = s^2 + 116.475s + 5642.395$,

$b_1(s) = 57.5\,s - 7520$, $a_1(s) = -19.77s - 40.005$,

$b_2(s) = -3638.15s + 1251135$, $a_2(s) = 58.555\text{s} + 5557.6$

q_i - uncertain coefficients

The robust PID controller has been designed by Neymark's D-partition method for 4 vertices of the polytopic system

$$G_R(s) = K\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Controller 1:

The required degree of stability: $\alpha = 0.3$

where the gain $K = 0.01$, the integration time constant $T_i = 0.333\ [s]$ and the derivative time constant $T_d = 0.003\ [s]$. Stability has been verified for each edge of the box **Q** by Bialas Theorem and all eigenvalues of the Bialas matrices were not nonpositive real. Therefore, the closed-loop with polytopic systems and robust controller is stable and the achieved degree in 4 vertices is $\alpha = 2.08$.

Controller 2:

The required degree of stability: $\alpha = 0.1$

where the gain $K = 0.015$, the integration time constant $T_i = 0.75\ [s]$ and the derivative time constant $T_d = 0.00333\ [s]$. Stability has been verified for each edge of the box **Q** by Bialas Theorem and all eigenvalues of the Bialas matrices were not nonpositive real. Therefore, the closed-loop with polytopic systems and robust controller is stable and the achieved degree in 4 vertices is $\alpha = 1.03$.

Controller 3:

The required degree of stability: $\alpha = 0$

where the gain $K = 0.04$, the integration time constant $T_i = 2\ [s]$ and the derivative time constant $T_d = 0.0025\ [s]$. Stability has been verified for each edge of the box **Q** by Bialas Theorem and all eigenvalues of the Bialas matrices were not nonpositive real. Therefore, the closed-loop with polytopic systems and robust controller is stable and the achieved degree in 4 vertices is $\alpha = 0.45$.

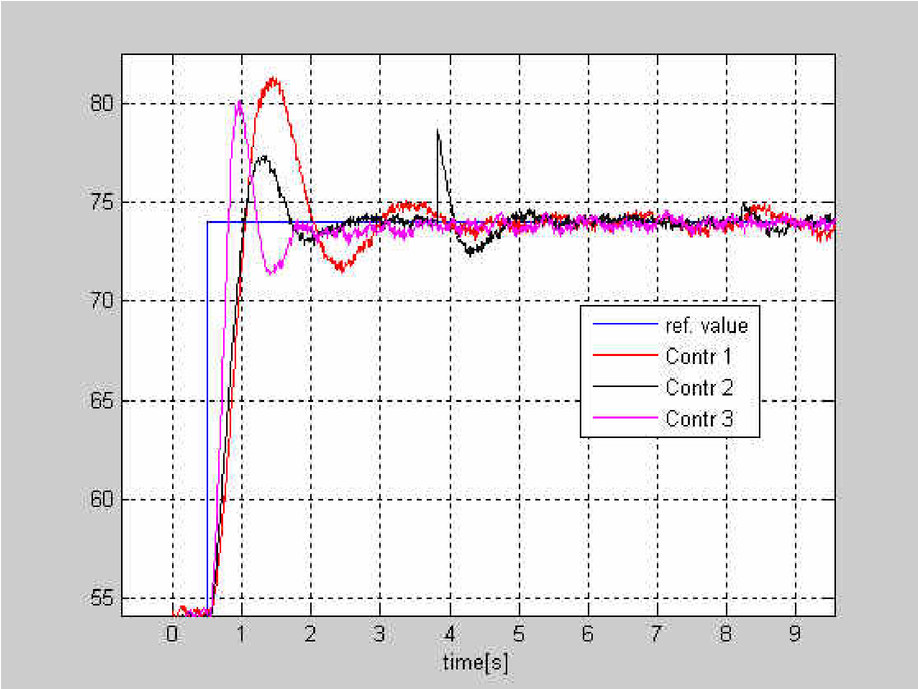

Figure 2: Step responses in the first working point

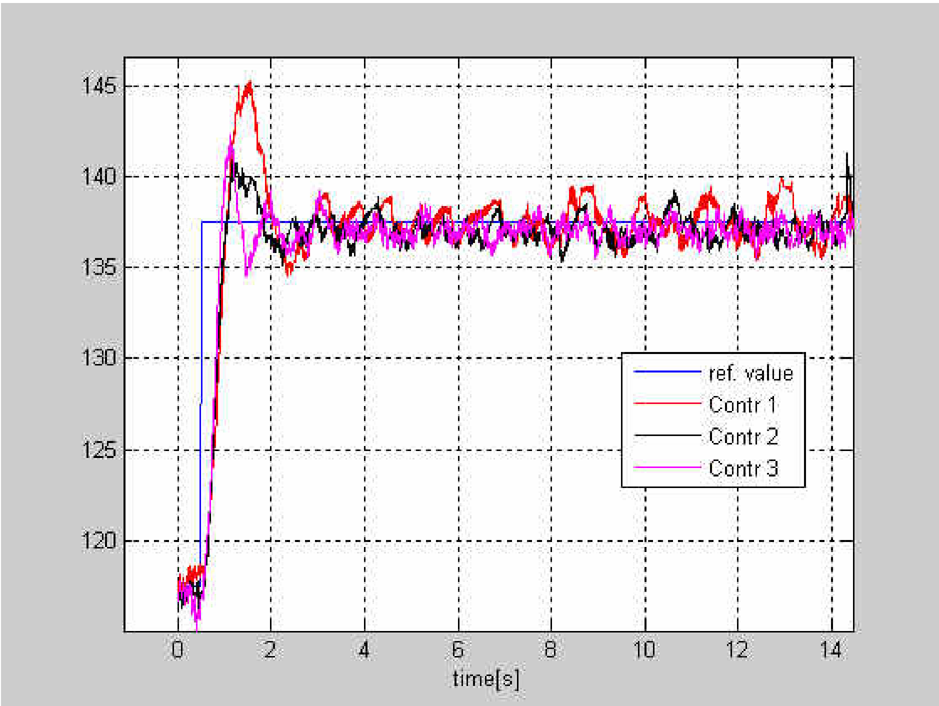

Figure 3: Step responses in the second working point

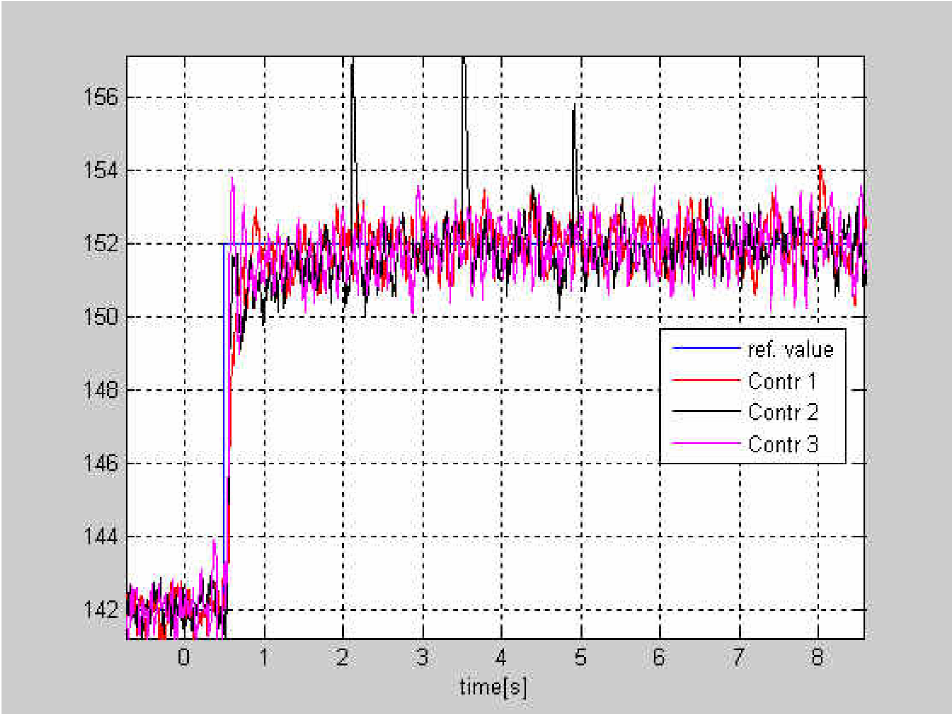

Figure 4: Step responses in the third working point

Figures 2, 3 and 4 show the step responses designed robust controller in three working points.

7 Conclusion

The main aim of this paper has been to design a robust controllers with different stability degree for MSS (Modular Servo System). Edge Theorem method which guarantee the required closed-loop stability degree has been used. It is a paradoxical that in this case the best controller is controller with lowest stability degree.

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Acknowledgments

The work has been supported by the grant agency VEGA no. 1/3841/06.

Ing. Mária Hypiusová, PhD.:

Institute of Control and Industrial Informatics, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava, Slovak Republic, Ilkovičova 3, 812 19 Bratislava, E-mail: maria.hypiusova@stuba.sk

Ing. Jakub Osuský

Institute of Control and Industrial Informatics, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava, Slovak Republic, Ilkovičova 3, 812 19 Bratislava, E-mail: jakub.osusky@stuba.sk

Ing. Slavomír Kajan, PhD.:
Institute of Control and Industrial Informatics, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava, Slovak Republic, Ilkovičova 3, 812 19 Bratislava, E-mail: slavomir.kajan@stuba.sk