

# VECTOR CONTROL OF TWO-PHASE INDUCTION MOTOR

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## Abstract

A general model suitable for Indirect Rotor-Field-Oriented Control (IRFOC) system of the two-phase induction machines including a relatively simple and effective decoupling scheme is proposed in the paper. However, model asymmetry in TPIM causes extra coupling between two stator windings. To use the IRFOC, the asymmetry must be eliminated by using an appropriate variable changing. In the mathematical model the machine main and auxiliary winding are represented in the stationary reference frame. Each phase of induction motor is fed independently by one-phase full-bridge inverter. A computer simulation of the IRFOC for two-phase induction motor drive is realized and simulation results presented.

## 1 Introduction

Two-phase induction motors (TPIMs) are one of widely used motors. The availability of low-cost static converters makes possible the economic use of energy and improvement of the quality of the electromagnetic torque in the TPIM. Conventionally, these machines are fed from single-phase ac mains supply with start-up capacitor. Indirect Rotor-Field-Oriented Control (IRFOC) is modern technique for high-performance control of PWM inverter fed TPIM. To achieve a variable speed operation a power electronics inverter can be used. The cost reduction of the semiconductor switches and efficient use of energy, even for low-power application, has stimulated the investigation of different two-phase motor drive schemes [1], [2] and [3]. In these schemes the single-motor drive schemes, without startup and running capacitor, is treated as an asymmetric two-phase machine. This paper investigates the use of the scheme in Fig.1 to feed independently each phase of TPIM. The field oriented strategy which will be described is based on the papers [4]-[7].

The purpose of this paper is to examine in simulation way the operation of a two-phase induction motor under IRFOC strategy.

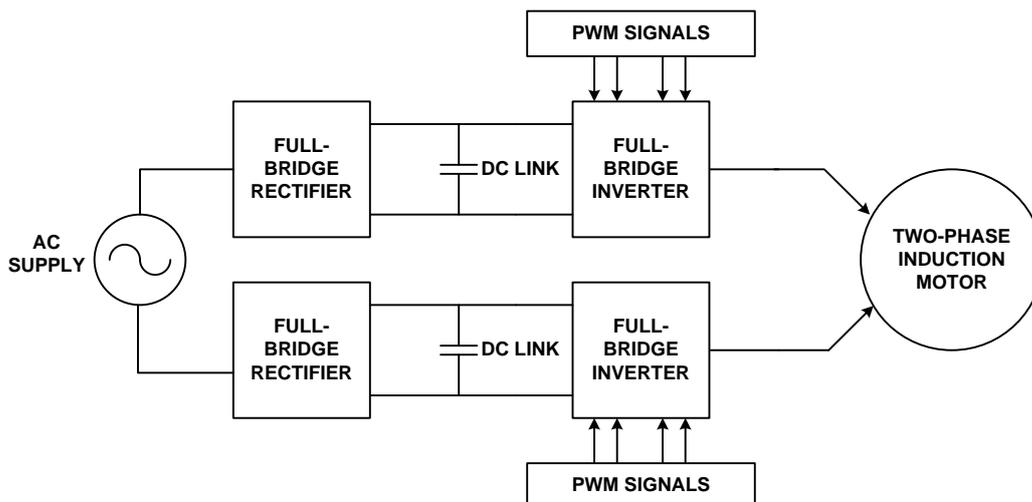


Figure 1: Two one-phase inverter for feeding two-phase induction motor

## 2 TPIM model

The stator voltage equations of a Two-Phase Induction Motor in a stationary reference frame can be represented by:

$$V_{sd}^s = R_{sd}^s i_{sd}^s + \frac{d\phi_{sd}^s}{dt} \quad (1)$$

$$V_{sq}^s = R_{sq}^s i_{sq}^s + \frac{d\phi_{sq}^s}{dt} \quad (2)$$

$$0 = R_r^s i_{rd}^s + \frac{d\phi_{rd}^s}{dt} + \omega \phi_{rq}^s \quad (3)$$

$$0 = R_r^s i_{rq}^s + \frac{d\phi_{rq}^s}{dt} - \omega \phi_{rd}^s \quad (4)$$

The stator and rotor-flux linkage components are given by:

$$\phi_{sd}^s = L_{sd}^s i_{sd}^s + M_{srd} i_{rd}^s \quad (5)$$

$$\phi_{sq}^s = L_{sq}^s i_{sq}^s + M_{srq} i_{rq}^s \quad (6)$$

$$\phi_{rd}^s = L_r i_{rd}^s + M_{srd} i_{sd}^s \quad (7)$$

$$\phi_{rq}^s = L_r i_{rq}^s + M_{srq} i_{sq}^s \quad (8)$$

The mechanical equation is:

$$J \frac{d\omega}{dt} + f_r \omega = n_p (T_e - T_l) \quad (9)$$

Where the electromagnetic torque is expressed by:

$$T_e = n_p (M_{srq} i_{sq}^s i_{rd}^s - M_{srd} i_{sd}^s i_{rq}^s) \quad (10)$$

In order to assure the control and the parameter estimation, the TPIM model should be presented in state space frame.

$$\frac{d[X]}{dt} = [A][X] + [B][U] \quad (11)$$

The TPIM state space model in a stationary reference frame is given by:

$$\frac{d}{dt} \begin{bmatrix} i_{sd} \\ i_{sq} \\ \phi_{rd} \\ \phi_{rq} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sigma_d \tau_{sd}} - \frac{1-\sigma_d}{\sigma_d \tau_r} & 0 & \frac{1-\sigma_d}{\sigma_d M_{srd} \tau_r} & \frac{1-\sigma_d}{\sigma_d M_{srd}} \omega \\ 0 & -\frac{1}{\sigma_q \tau_{sq}} - \frac{1-\sigma_q}{\sigma_q \tau_r} & \frac{1-\sigma_q}{\sigma_q M_{srq}} \omega & \frac{1-\sigma_q}{\sigma_q M_{srq} \tau_r} \\ \frac{M_{srd}}{\tau_r} & 0 & -\frac{1}{\tau_r} & -\omega \\ 0 & \frac{M_{srq}}{\tau_r} & \omega & -\frac{1}{\tau_r} \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \\ \phi_{rd} \\ \phi_{rq} \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma_d} & \frac{1}{L_{sd}} & 0 \\ 0 & \frac{1}{\sigma_q} & \frac{1}{L_{sq}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{sd} \\ V_{sq} \\ 0 \\ 0 \end{bmatrix} \quad (12)$$

in which:

$$\sigma_d = 1 - \frac{M_{srd}^2}{L_{sd}^s L_r} \quad \sigma_q = 1 - \frac{M_{srq}^2}{L_{sq}^s L_r} \quad \tau_{sd} = \frac{L_{sd}}{R_{sd}} \quad \tau_{sq} = \frac{L_{sq}}{R_{sq}} \quad \text{and} \quad \tau_r = \frac{L_r}{R_r}$$

### 3 Indirect Rotor-Flux-Oriented Control

Note that (1) - (10) are general equations for the two-phase machine. They may represent either a symmetrical or an asymmetrical model, it can be seen from (10) that the machine produces torque oscillations if the d and q are balanced. The term balanced denotes that the variables d and q are sinusoidally phase shifted by  $90^\circ$  and with the same amplitude. This asymmetry is due to the unequal resistances and inductances of the main and auxiliary windings. However, to use the field orientation control of unbalanced two-phase induction motor, the asymmetry must be eliminated using an appropriate variable changing as presented in [4]:

$$i_{sd}^s = i_{sd1}^s \quad (13)$$

$$i_{sq}^s = k i_{sq1}^s \quad (14)$$

where:  $k = \frac{M_{srd}}{M_{srq}}$ .

By substituting the variables  $i_{sd}$  and  $i_{sq}$  for the auxiliary variables  $i_{sd1}$  and  $i_{sq1}$  into (10), the torque can be expressed by:

$$T_e = \frac{n_p}{L_r} M_{srd} (i_{sq1}^s \phi_{rd}^s - i_{sd1}^s \phi_{rq}^s) \quad (15)$$

The new expression of the electromagnetic torque is similar to the one of a symmetrical machine. The relationship between rotor-flux components and stator currents in the stationary reference frame are deduced from (3),(4),(7),(8),(13) and (14) equations

$$\frac{d\phi_{rd}^s}{dt} = \frac{M_{srd}}{\tau_r} i_{sd1}^s - \frac{1}{\tau_r} \phi_{rd}^s - \omega \phi_{rq}^s \quad (16)$$

$$\frac{d\phi_{rq}^s}{dt} = \frac{M_{srd}}{\tau_r} i_{sq1}^s - \frac{1}{\tau_r} \phi_{rq}^s + \omega \phi_{rd}^s \quad (17)$$

The vector model is defined from (16) and (17) by rearranging the variables in the vector form. If this vector model is written for an arbitrary reference frame (denoted by superscript a), which is  $\delta_a$  rad away from phase d of the stator, then

$$\frac{d\phi_r^a}{dt} = -\frac{1}{\tau_r} \phi_r^a - j(\omega_a - \omega) \phi_r^a + \frac{M_{srd}}{\tau_r} i_{s1}^a \quad (18)$$

And  $\omega_a = \frac{d\delta_a}{dt}$  is the speed of the arbitrary reference frame. The variables in the arbitrary reference frame are calculated from the variables in the stator reference frame through the following equations:

$$\phi_r^a = \phi_{rd}^a + j\phi_{rq}^a = (\phi_{rd}^s + j\phi_{rq}^s) e^{-j\delta_a} \quad (19)$$

$$i_{s1}^a = i_{sd1}^a + j i_{sq1}^a = (i_{sd1}^s + j i_{sq1}^s) e^{-j\delta_a} \quad (20)$$

Based on the vector model given by (15) and (18) it is possible to apply the field oriented principles to control the rotor flux and electromagnetic torque of two-phase machine. For that, the rotor flux reference frame (denoted by the superscript rf) is chosen and, consequently,  $\phi_{rd}^{rf} = \phi_r^a$  and  $\phi_{rq}^{rf} = 0$ . Then torque and flux-current equations in the rotor-flux reference frame can be obtained from (18), that is,

$$\frac{M_{srd}}{\tau_r} i_{sd1}^{rf} = \frac{\phi_r}{\tau_r} + \frac{d\phi_r}{dt} \quad (21)$$

$$\frac{M_{srd}}{\tau_r} i_{sq1}^{rf} = \omega_{sl} \phi_r \quad (22)$$

Using (15) equation, the expression for the torque can be calculated by:

$$T_e = \frac{n_p}{L_r} M_{srd} i_{sq1}^{rf} \phi_r \quad (23)$$

Where  $\omega_{sl} = \omega_s - \omega$  is the slip frequency,  $\omega_s = \frac{d\delta_{rf}}{dt}$  and  $\delta_{rf}$  are respectively the frequency and the position of the rotor flux vector. Then,  $i_{sd1}^{rf}$  controls the rotor flux and  $i_{sq1}^{rf}$  controls the electromagnetic torque. As an example, Fig.2 shows the block diagram of indirect rotor-field-oriented control scheme, which has been adapted for the two-phase machine. In this diagram  $T_e^*$  and  $\phi_r^*$  represent the reference electromagnetic torque and amplitude of the rotor flux, respectively. Block  $e^{j\delta_{rf}}$  performs the coordinate transformation from the reference frame aligned along with the rotor-flux vector to the stationary reference frame. Furthermore,  $i_{sd1}^{s*}$  and  $i_{sq1}^{s*}$  represent the reference currents supplied to the PI stator current controllers, which must be imposed on the machine windings. It is seen that the two current controllers provide control voltages  $V_{sd}$  and  $V_{sq}$ . These voltages are supplemented by decoupling voltages  $E_d$  and  $E_q$  respectively to produce voltages commands. Block PWM generates the gate signal according voltage commands for each transistor of the converters and each phase of TPIM is fed by independent full-bridge inverter. Speed sensor  $S_\omega$  gives information about rotor angular speed.

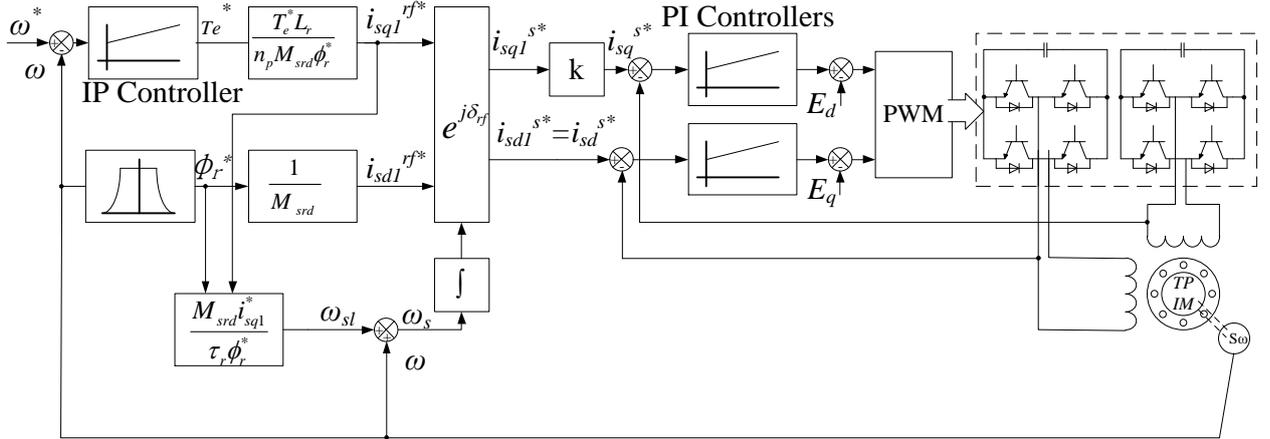


Figure 2: Block scheme of the indirect rotor-flux-oriented control

## 4 Simulation results

Simulation experiment was realized in program Matlab/Simulink. The parameters of the simulation are shown in Table.1:

Table 1: PARAMETERS OF THE SIMULATION

$R_{sd} = 2.4 \Omega$	$R_{sq} = 5.66 \Omega$	$R_r = 6.161 \Omega$	$L_{sd} = 0.0909 \text{ H}$
$L_{sq} = 0.115 \text{ H}$	$L_r = 0.0915 \text{ H}$	$M_{srd} = 0.0829 \text{ H}$	$M_{srq} = 0.099 \text{ H}$
$T_1 = 5 \text{ Nm}$	$n_p = 2$	$J = 2.63 \cdot 10^{-4} \text{ Kg.m}^2$	$f_r = 2.026 \cdot 10^{-4} \text{ N.m.s.rad}^{-1}$

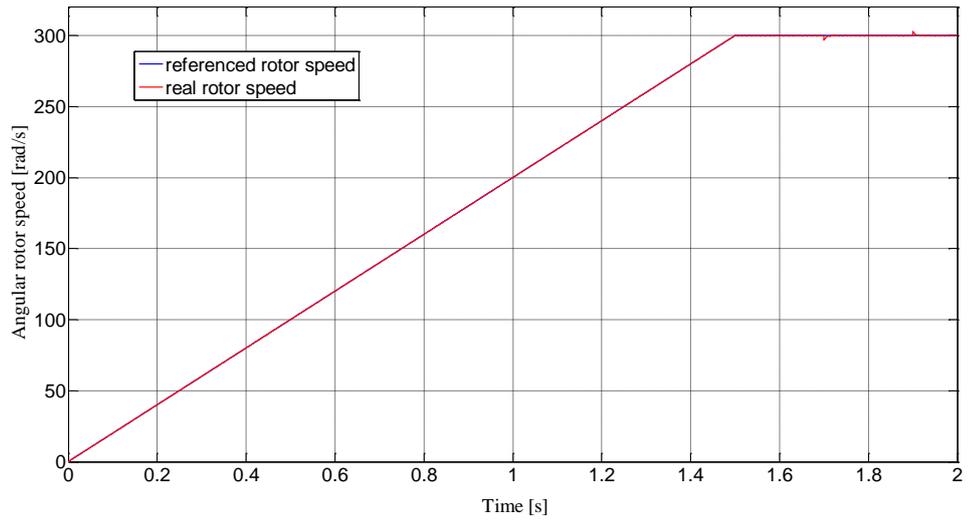


Figure 3: Referenced and real angular rotor speed

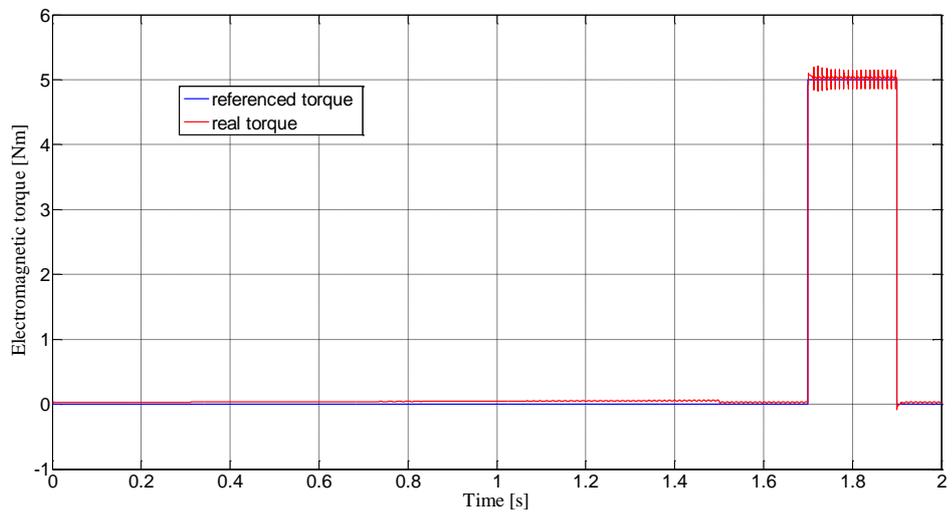


Figure 4: Referenced and real electromagnetic torque

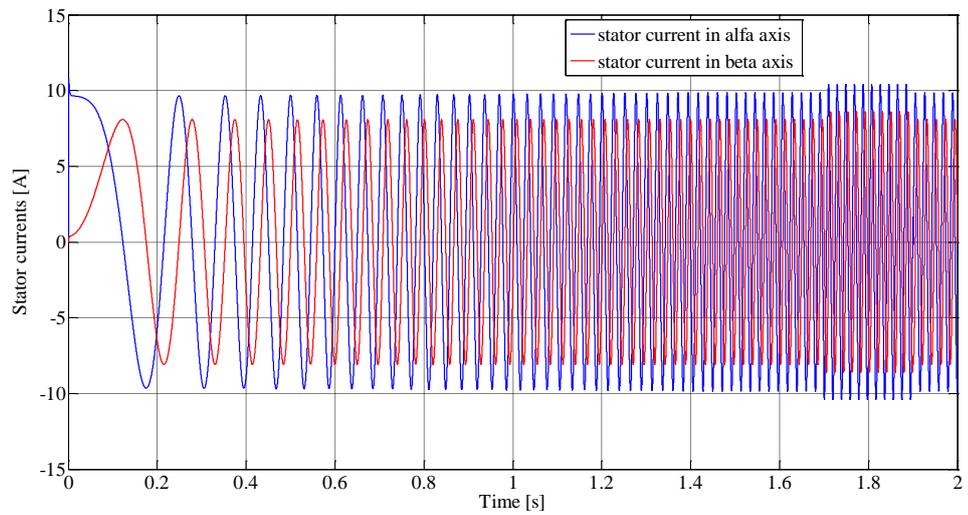


Figure 5: Stator currents in stationary reference frame

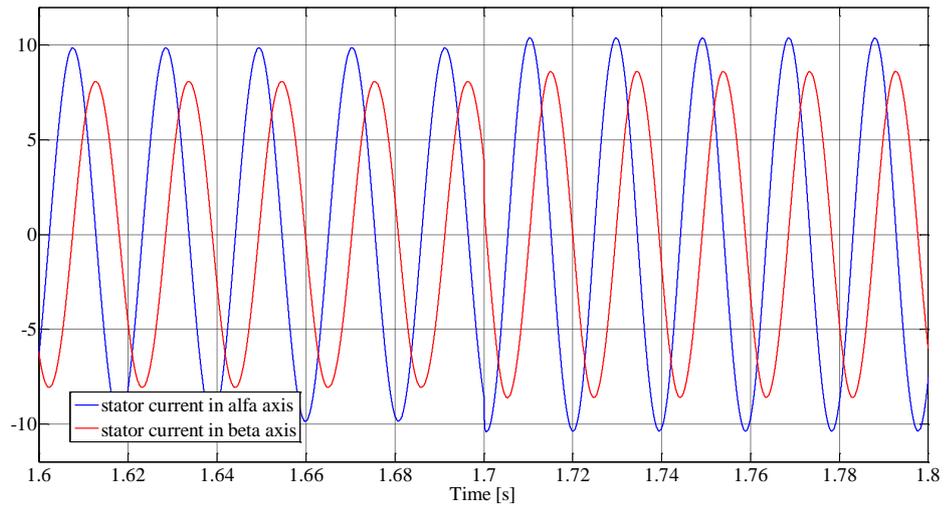


Figure 6: Detail of stator currents in stationary reference frame

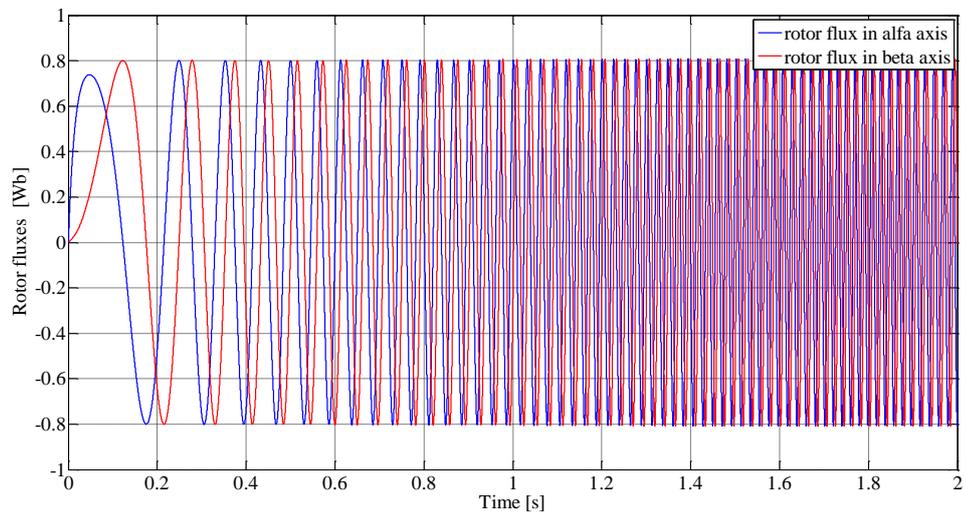


Figure 7: Rotor fluxes in stationary reference frame

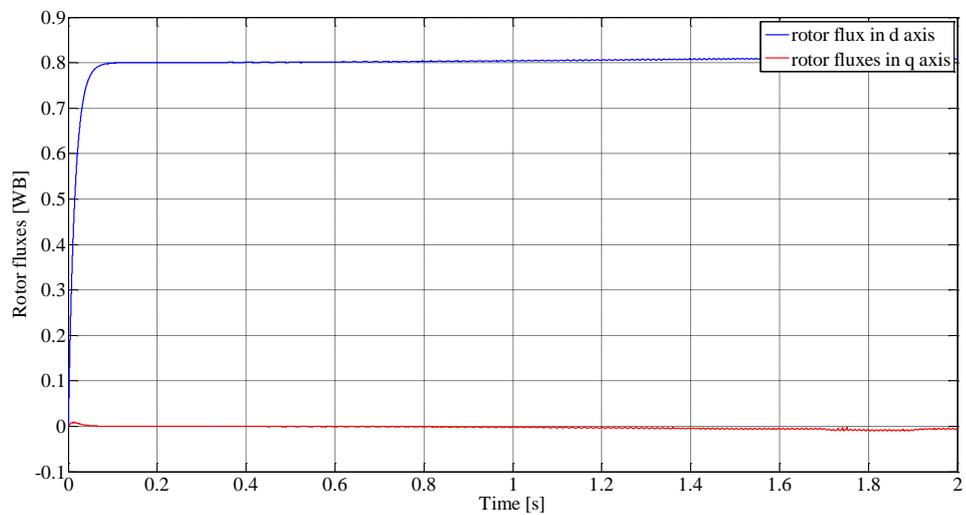


Figure 7: Rotor fluxes in synchronous reference frame

The experiment have been performed for a starting at no load from zero to rated speed  $\omega = 300$  rad/s with prescribed behavior, linear increase of speed for one and a half second as shown in Fig.3, which presents referenced and real speed. Load torque (Fig.4) applied in 1.7s and derating the motor in 1.9s caused only small overshoot of the rotor speed. Stator currents (Fig.5 and detail Fig.6), which supplied the TPIM had different amplitude, what is dependent on the different parameters of main and auxiliary winding of the machine. From Fig.7 we can see that the rotor fluxes in stationary reference frame are equal in amplitude, so the asymmetry of the TPIM was eliminated. From Fig.8, the reference d-axis rotor flux is kept constant at 0.8Wb while the q-axis rotor flux remains null, so it is clear that the rotor fluxes is aligned with the d-axis. As shown in the simulation, the proposed control algorithm has an improved and robust performance.

## 5 Conclusion

This paper makes a contribution to the issue of simulation IRFOC of two-phase induction motor drive. These results were satisfactory and the proposed IP controller gives the system good performances and good dynamic behavior.

## Acknowledgement

The authors wish to thank for the financial support to VEGA1/0470/09 and Slovak Research and Development Agency APVV project No. APVV-0138-10. Also the authors want to thank for the technical support to STMicroelectronics and PPI Adhesive Products.

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## Nomenclature

$V_{sd}, V_{sq}$	d, q axis stator voltage components
$i_{sd}, i_{sq}$	d, q axis stator current components
$V_{rd}, V_{rq}$	d, q axis rotor voltage components
$i_{rd}, i_{rq}$	d, q axis rotor current components
$\phi_{sd}, \phi_{sq}$	d, q axis stator flux components
$\phi_{rd}, \phi_{rq}$	d, q axis rotor flux components
$R_{sd}, R_{sq}$	stator winding resistances
$R_r$	rotor resistance
$L_{sd}, L_{sq}$	stator self-inductances
$L_r$	rotor self-inductance
$M_{srd}, M_{srq}$	mutual inductances
$T_e, T_l$	electromagnetic and load torque
$s=d/dt$	differential operator
$\omega_s, \omega$	synchronous and rotor angular speed
$\omega_{sl}$	slip angular speed
$n_p$	pole-pair number
$f_r$	friction coefficient
$J$	total inertia
$\sigma_d, \sigma_q$	leakage coefficient
$\tau_{sd}, \tau_{sq}, \tau_r$	stator and rotor time constants

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