APPLICATION FOR VERY SENSITIVE DIGITAL HOLOGRAPHIC VIBRATION ANALYSIS

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Abstract

   This paper describes MATLAB GUI application, which processes data obtained by newly developed method for vibration analysis. The method is based on the frequency-shifted time average digital holographic interferometry employing the principle of phase shifting. This approach allows for a significant noise reduction, which results in the high sensitivity of measurements. The method makes it possible to measure the vibrations with amplitudes in the nanometer range over the whole measured surface.

1   Principle of digital holographic method for vibration analysis

   For measurement of small amplitudes of oscillating surfaces it is essential to use a non-invasive and very precise. Optical based methods, where the measurement scale is usually wavelength of the used light source, appear to be very appropriate. Although some types of vibrometers or single-point intrerferometric methods provide precise value of out-of-plane deviations, it is beneficial to have possibility of the simultaneous displacement measurement over the whole-field. Digital holography matches the above mentioned criteria. It is whole-field and non-invasive technique with widely spread applications in industry and research. A digital hologram is done, if interference pattern of a reference wave and the wave reflected from the object is recorded by a digital recording medium (CCD or CMOS sensor). The resulting hologram is electronically recorded and stored. The diffusely reflecting object is located at a distance d far from the CCD. For digital reconstruction the recorded hologram $h(\xi, \eta)$ is multiplied with a numerical model of reference wave $r(\xi, \eta)$. The field in the image plane U is calculated by the Sommerfeld formula which describes the diffraction of a light wave at the hologram in distance d from the hologram. The Sommerfeld integral can be solved by Fresnel approximation which in discrete finite form can be written as:

$$U(n,m) = e^{j\pi\lambda d\left(\left(\frac{n}{N\Delta\xi}\right)^2 + \left(\frac{m}{M\Delta\eta}\right)^2\right)} \times \qquad (1)$$

$$\times \sum_{k=1}^{N}\sum_{l=1}^{M} h(k\Delta\xi, l\Delta\eta) r(k\Delta\xi, l\Delta\eta) e^{\frac{j\pi}{\lambda d}((k\Delta\xi)^2 + (l\Delta\eta)^2)} e^{-j2\pi\left(\frac{kn}{N} + \frac{lm}{M}\right)},$$

where j $= \sqrt{-1}$ and λ is the wavelength of the light. The stored hologram consists of $N \times M$ discrete values with pixel size $\Delta\xi \times \Delta\eta$. This formula can be solved effectively by the FFT-algorithm. Intensity I and phase φ distributions is then determined by

$$I(n,m) = |U(n,m)|^2$$
$$\varphi(n,m) = arctan\frac{Im\{U(n,m)\}}{Re\{U(n,m)\}}. \qquad (2)$$

   The experimentally easiest and the most often applied holographic method for vibration analysis is the time average holographic interferometry which was introduced by Powell and Stetson [1]. Consider a harmonically oscillating diffusely reflecting object in position p: $d(p,t) = d(p)sin\omega t$, where ω is the circular frequency and $d(p)$ is the amplitudeof vibration and such an oscillating object is recorded holographically using an exposure time much longer compared to the period of the vibration of the object $T \gg 2\pi/\omega$. The resulting intensity in the reconstructed image at position p is: $I(p) = J_0^2(d(p).e(p))$,where $e(p)$ is the sensitivity vector given by geometry of the holographic arrangement and J_0 is the first kind zero-order Bessel function, see Fig. 1. Because there is a steady state at the nodes of the vibration modes, the intensity image exhibits the maximal intensity, $J_0(0) = 1$, in these points. Further there are dark fringes where $d(p).e(p)$ equals the arguments of the zeros of

the zero-order Bessel function of the first kind. The bright fringes between these zeros which do not correspond to nodal lines exhibit less intensity compared with the zero fringe. Thus the nodal lines can easily be identified. The simplest example is an out-of-plane vibrating surface and a holographic arrangement with illumination and observation normal to the surface. If each point can be assumed to oscillate only in the normal direction to the surface $d(p) = (0,0, d_Z(p))$, then $e(p) = 4\pi/\lambda$ and the amplitudes at the centers of dark fringes are easily calculated by $d_Z(p) = b_m \lambda/4\pi$, where b_m is the m-th zero of J_0 and λ is wavelength of the laser light.

Unfortunately, for amplitudes smaller than approximately one tenth of the wavelength of used laser light, the time-average method reaches its limit of minimum measurable amplitude. This problem was solved by frequency shifted holographic interferometry introduced by Aleksoff [2]. In frequency shifted holography the frequency of the reference wave is modulated by an integer multiple $n\omega$ of the object vibration frequency ω: $f_{ref}(t) = e^{jn\omega t}$. If the object wave U_P and the reference wave U_R is modulated by f_{ref}, the resulting intensity in the hologram plane at time t is proportional to: $I(t) = |U_P + f_{ref}(t)U_R|^2$. Using solution of interference holographic equation and properties of Bessel functions we obtain:

$$I(p) = |U_{Jn}|^2 = J_n{}^2(d(p).e(p)). \tag{3}$$

Frequency shifted holography is used to increase the sensitivity for vibrations with small as well as with large amplitudes. In time average holography with no modulation $J_0{}^2(0)$ is unity and has zero slope (the derivate is constant). Therefore for the small amplitudes, it is difficult to determine significant intensity variation in the bright field, see Fig. 1 (left). On the other hand $J_1{}^2(0)$ has a positive slope in the dark field yielding visible intensity variations even for small amplitudes, see Fig. 1 (right). This is the key of the method. The smallest detectable amplitude was estimated by Ueda, Miida and Sato [3] as $d(p) \approx \lambda/3500$.
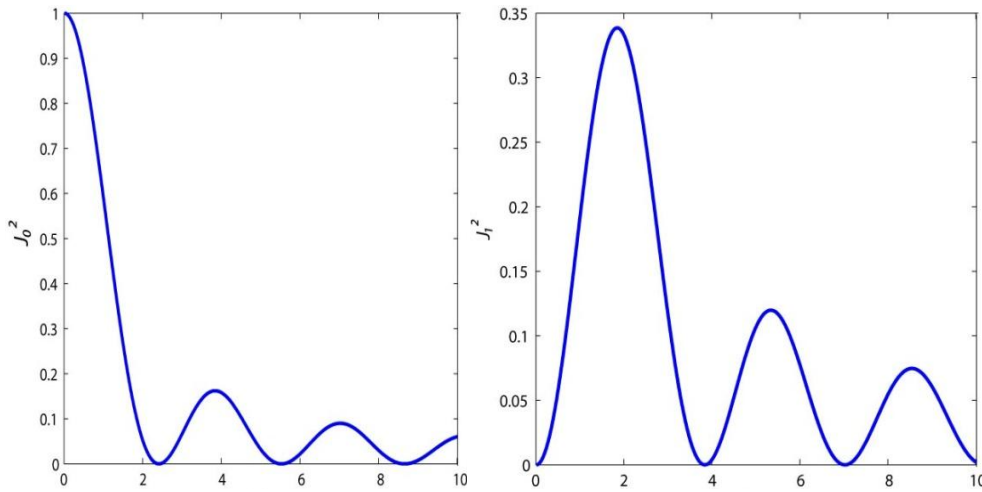

Figure 1: $J_0{}^2$ Bessel function (left) and $J_1{}^2$ Bessel function

2 Data Processing

The method for very sensitive vibration analysis is based on the application of frequency shifted digital holography with the employment of phase shifting principles [4]. From the measurement, a set of phase-shifted digital holograms is obtained. The sequence of *M* digital holograms is recorded and processed by formula $h = \frac{1}{M}\sum_{m=0}^{M-1} h_m e^{2\pi\frac{m}{M}}$ to get a filtered digital hologram *h*. Afterwards the processed hologram is reconstructed using Fresnel transform (1) and the intensity distribution is calculated with use of formula (2). The interferogram reconstructed from the processed hologram h has a much higher signal to noise ratio and strongly suppressed DC term compared to the interferogram reconstructed from a single hologram. To suppress speckle noise in interferograms, generated by coherent illumination, some advanced filtering methods can be used [5]. Low overall

noise is a key factor to get a very high resolution of the method. Since the reflectance of the measured surface is not uniform in the whole surface, the reference interferogram with no frequency shift of reference wave in steady state must also be recorded. Such an interferogram represents J0 intensity distribution of the steady state. The reference interferogram UJ0 is used to normalize the interferogram UJ1 reconstructed from digital hologram with frequency shifted reference wave: $U_{J1norm} = U_{J1}/U_{J0}$. A necessary condition for the normalization is a uniform response of the recording medium during all records. Modern digital cameras permit the uniform record. On the other hand, typical analogue recording mediums being used in holography do not have this property and therefore are not suitable for this method. After the operation of normalization we get a normalized version of (3): $U_{J1norm}(p) = J_1(d(p).e(p))$. In our experimental setup the sensitivity vector is approximately $e(p) = 4\pi/\lambda$ so the out-of-plane vibrations amplitude d in every surface point p can be directly calculated by inversion of

$$U_{J1norm}(p) = J_1(d(p)4\pi/\lambda). \tag{4}$$

For the inverse transfer function it is necessary to determine the out-of-plane vibrations amplitude corresponding to J_{1MAX}. The normalization of U_{J1} permits to use well defined relations between J0 and J1 functions:

$$J_{1MAX} = 0,582 \; and \; J_0(0) = J_1(1,84). \tag{5}$$

Therefore we can derive the maximum amplitude: $d_M = 1,84\,\lambda/4\pi$. The transfer function for typical coherent source $\lambda = 532$ nm is displayed in Fig. 3. This maximum amplitude d_M is also limiting value for maximal measured amplitude by this method without the risk of ambiguities. For greater amplitudes, standard time average digital holography represented by J0 function serves perfectly.
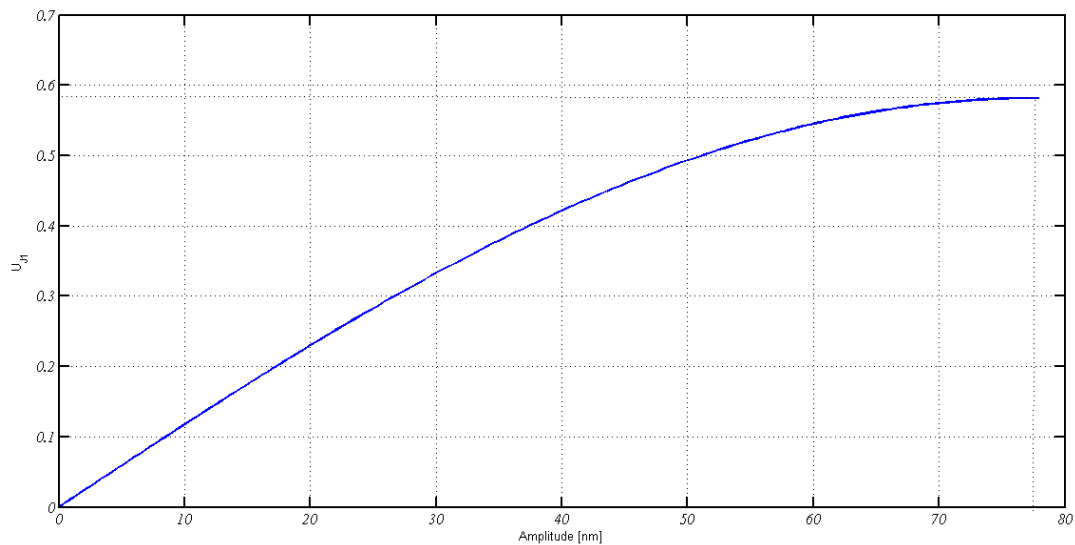

Figure 2: The transfer function.

3   MATLAB GUI application

For the data processing as it was described in previous chapter, a MATLAB GUI application serves. The whole application is divided into four logical panel tabs: Phase-Shifting, Reconstruction, Vibration Analysis and Visualization. The structure of the MATLAB GUI is shown in Fig. 2.
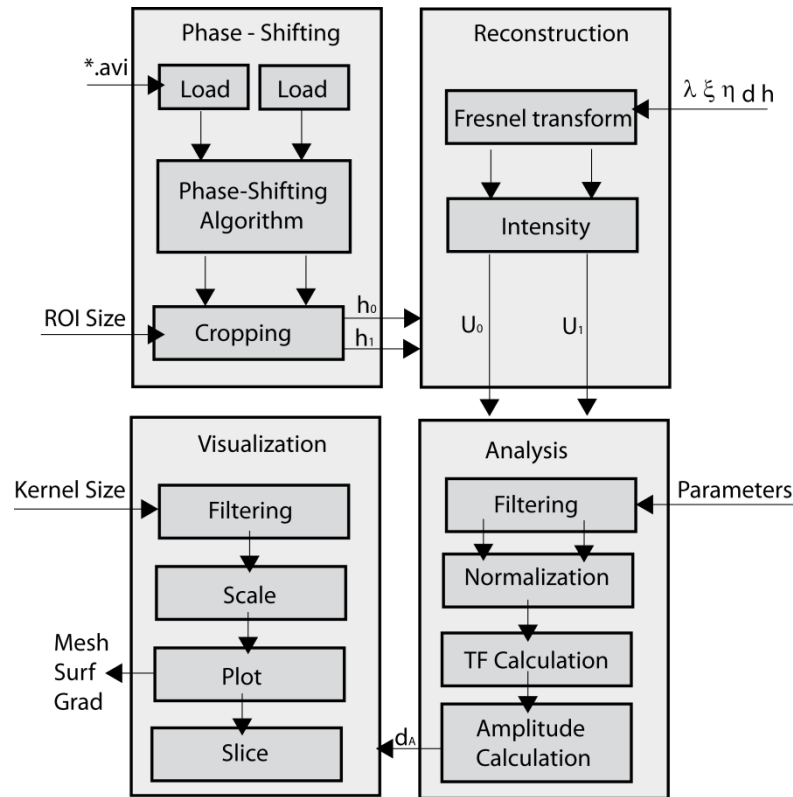

Figure 3: Structure and data flow of the application.

Phase-Shifting

At first a set of digital holograms must be loaded from a file. The set of digital holograms h_m is usually stored in computer memory as an .avi file. The button "Load" displays a modal dialog box that lists files in the current folder and enables us to select or enter the name of a file. The current folder path is saved, which brings more comfort for repeated using of "Load" button callback, because we do not have to search the current folder again. If we chose required .avi file, a sequence of M digital holograms is loaded and processed by phase-shifting formula. Now the digital hologram is ready for processing and can be displayed in its current size using the "Display" button. Statistical analysis of the digital hologram histogram can be displayed by pressing the "Histogram" button. It helps us to decide if the shutter of the digital camera used for digital holographic recording was set optimally.

In reconstruction of wave fields the Fast Fourier Algorithm, which needs 2^n values, is used. Therefore, it is recommended to crop the digital holograms using 2^n squared region. The button "Crop" serves to this effect. If the check box "Crop - centre" is selected, the cropped image is the squared array placed in the centre of initial image. Its size can be set by "edit text" "ROI Size". If the check box "Crop - centre" is not selected, the Crop Image Tool with region of interest (its size is also given by "edit text" "ROI Size" value) is displayed. The region is a moveable, resizable rectangle which we can position interactively using the mouse. Then the cropping is done by double clicking inside the region. The centre cropping in the majority of cases is sufficient for processing of a digital hologram.

Reconstruction

The reconstructed wave field results from the formula (1). To simplify and accelerate the calculation of the reconstruction equation, the inverse discrete Fourier transform can be used to get:

$$g(n\Delta x, m\Delta y) = \frac{e^{-j\frac{2\pi}{\lambda}d}}{\lambda d} \mathcal{F}^{-1}\left\{h(k\Delta\xi, l\Delta\eta)r^*(k\Delta\xi, l\Delta\eta)e^{\frac{j\pi}{\lambda d}((k\Delta\xi)^2+(l\Delta\eta)^2)}\right\},$$

where $\mathcal{F}^{-1}$ is the inverse Fourier transform, d is CCD – object distance, λ is the wavelength of used laser and $\Delta\xi \times \Delta\eta$ is the CCD pixel size in orthogonal directions. All these values must be set by the user.

Vibration Analysis

In this section, firstly, the intensity fields reconstructed from digital holograms are filtered. The "Filter" button starts the filtering process. Since the measured surface is diffusively reflecting, the speckle noise is present and must be reduced. The speckle noise is suppressed by reduction of field size in the following way [8]. From every 2x2 matrix, pixel, where the intensity becomes maximum, is selected. Then the reduced intensity field is filtered by edge preserving Lee filter [7]. After the "Amplitude Calculation" button click, the normalization process is done, the transfer function and the amplitude field are calculated.

Visualization

In this tab panel, functions for processing and displaying of resulting amplitude distribution are available. Basicaly, the amplitude field can be displayed. It is done after clicking the "Amplitude distribution" button. If the radio button "Mesh" is selected, a colored wireframe mesh is drawn, where the color is proportional to the surface deformation is drawn. On the other hand, if the radio button "Surf" is selected a three-dimensional shaded surface is created. Both plots are partly displayed in the user interface and they can also be displayed in their own figure if the checkbox "Display in Figure" is checked. There is also a possibility of convolution filtering in the amplitude distribution. The matrix size *n* of the convolution kernel can be set by edit text „Squared Kernel Size"and filtering is done again by „Amplitude distribution" button click. So we obtain the smoothed version of the amplitude distribution. If the user sets the value in the edit text „Squared Kernel Size" to one, no filter will be applied. The user can also interpolate or decimate the deformation field. The decimation appears advantageous in case of large fields. There the pixels density is unnecessarily high. The size of the deformation field can be changed by edit text "Change array size". In the application also the axis ratio can be adjusted. This can be used to obtain better 3-D plot of deformation field. The colorbar scale can be adjusted manually or automatically according to minimal and maximal value of the deformation field. In measurement of the amplitude distribution we can also be interested in the direction of the greatest rate of increase of the scalar field – the gradient. The button "Amplitude Gradient" serves for this purpose. Simultaneously the level curve (a curve along which the function has a constant value) is plotted. Another useful function for results interpretation is "slice", which permits the user to plot function values along a certain line. After the "Slice" button click the image of amplitude distribution is displayed. There we can draw any line by a drag mouse. The line is resizable and draggable and finally the values of deformation are plotted along the line.

4 Conclusion

In this paper the basic principle of newly developed measuring method for measurement of very small amplitudes of oscillating surfaces was described. For processing of the measured data, the MATLAB GUI application comprising all the necessary operations was created and its basic functions were introduced. For declaration of success of this method, illustrative examples of results from piezoelectric transformers measurement is shown, see Fig 4b.
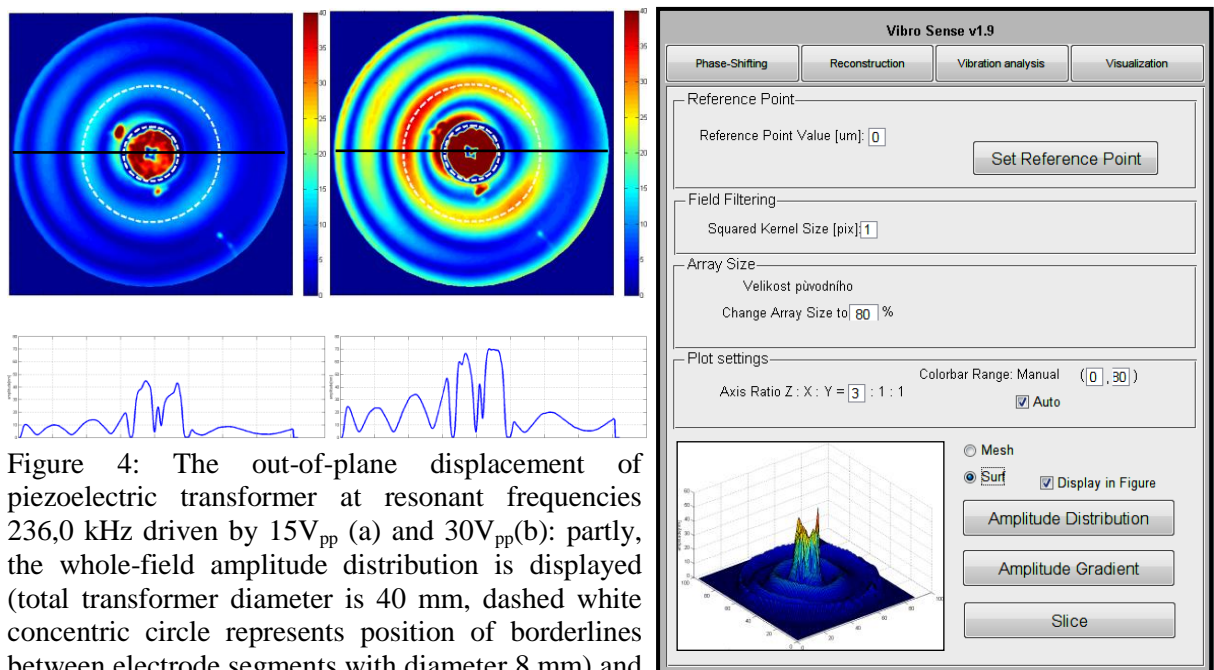
Figure 4: The out-of-plane displacement of piezoelectric transformer at resonant frequencies 236,0 kHz driven by 15V$_{pp}$ (a) and 30V$_{pp}$(b): partly, the whole-field amplitude distribution is displayed (total transformer diameter is 40 mm, dashed white concentric circle represents position of borderlines between electrode segments with diameter 8 mm) and the amplitude values along the black line are plotted bellow. The MATLAB GUI is also shown (c).

Acknowledgements

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