GENETIC AND ROBUST CONTROLLER DESIGN METHODS FOR UNCERTAIN SISO SYSTEMS

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Abstract

The paper deals with a genetic and robust controller design methods for uncertain SISO systems. The main idea of paper is to comparison of classical method for robust controller design (Edge Theorem and Small Gain Theorem) with genetic algorithm (GA). The first approach is accomplished with the Edge Theorem and the Neymark D-partition method for the affine model. The second controller design method is based on the Small Gain Theorem considering uncertain system model with additive uncertainty. For the both methods, the designer can specify a required closed-loop degree of stability. The genetic algorithm represents an optimisation procedure, where the costs function to be minimized comprises the closed-loop simulation of the control process and a selected performance index evaluation. Using this approach the parameters of the PID controller were optimised in order to become the required behaviour of the control process. The comparison of methods is illustrated by the robust controller design for two linear systems with uncertain parameters.

1 Introduction

For many real processes a controller design has to cope with the effect of uncertainties, which very often cause a poor performance or even instability of closed-loop systems. The reason for that is a perpetual time change of parameters (due to aging, influence of environment, working point changes *etc.*), as well as unmodelled dynamics. The former uncertainty type is denoted as the parametric uncertainty and the latter one the dynamic uncertainty. A controller ensuring closed-loop stability under both of these uncertainty types is called a robust controller. A lot of robust controller design methods are known from the literature [1], [2] in the time- as well as in the frequency domains.

The focus of this paper is to comparison of classical method for robust controller design (Edge Theorem and Small Gain Theorem) with genetic algorithm. The first approach is accomplished with the Edge Theorem and the Neymark D-partition method for the affine model. The second controller design method is based on the Small Gain Theorem considering uncertain system model with additive uncertainty. For the both methods, the designer can specify a required closed-loop degree of stability. The genetic algorithm represents an optimisation procedure, where the minimization of cost function comprises the closed-loop simulation of the control process and a selected performance index evaluation. Control performance indices corresponding to robust controllers designed for several required closed-loop stability degree are compared in several working points.

2 Robust controller design

2.1 Robust Controller design using the Edge Theorem

Consider an affine model of the plant in the form:

$$G(s)=\frac{B(s)}{A(s)}=\frac{b_0(s)+\sum_{i=1}^{p} b_i(s)q_i}{a_0(s)+\sum_{i=1}^{p} a_i(s)q_i} \tag{1}$$

where $q_i \in \langle \underline{q}_i, \overline{q}_i \rangle$ are uncertain coefficients. The coefficients depend linearly on uncertain parameter vector $\mathbf{q}^T = [q_1, ..., q_p]$; the parameters q_i vary within a p - dimensional box

$$\mathbf{Q} = \left\{ \mathbf{q} : q_i \in \langle \underline{q}_i, \overline{q}_i \rangle, i = 1, ..., p \right\}. \tag{2}$$

Consider $q_i = \underline{q}_i$ or $q_i = \overline{q}_i$; then we obtain 2^p transfer functions with constant coefficients; inserting them to the vertices of a p - dimensional polytope, the transfer function (1) describes a so-called *polytopic system*.

Consider the controller transfer function in the form

$$G_R(s) = \frac{F_1(s)}{F_2(s)} \tag{3}$$

where $F_1(s)$ and $F_2(s)$ are polynomials with constant coefficients. Then the characteristic polynomials with the polytopic system are

$$p(s, \mathbf{q}) = b_0(s) F_1(s) + a_0(s) F_2(s) + \sum_{i=1}^{p} q_i \left[b_i(s) F_1(s) + a_i(s) F_2(s) \right] \tag{4}$$

or in a more general form

$$p(s, \mathbf{q}) = p_0(s) + \sum_{i=1}^{p} q_i p_i(s), \ q_i \in \mathbf{Q} \tag{5}$$

Theorem 1 (Edge Theorem)

The polynomial family (5) is stable if and only if the edges of $\mathbf{Q}$ are stable.

A simple stability analysis method for families of polynomials (edges of $\mathbf{Q}$) is given in the following theorem.

Theorem 2 (Bialas)

Let $\boldsymbol{H}_n^{(a)}$ and $\boldsymbol{H}_n^{(b)}$ be the Hurwitz matrices corresponding respectively to

$$\begin{aligned} p_b(s) &= p_{b0} + p_{b1}s + p_{b2}s^2 + ... + p_{bn}s^n \quad p_{bn} > 0, \\ p_a(s) &= p_{a0} + p_{a1}s + p_{a2}s^2 + ... + p_{an}s^n \quad p_{an} > 0, \end{aligned} \tag{6}$$

The family of polynomials

$$p(s, Q) = \{ \lambda p_a(s) + (1 - \lambda) p_b(s), \quad \lambda \in [0, 1] \} \tag{7}$$

is stable if and only if:

1) $p_b(s)$ is stable

2) the matrix $\left(\boldsymbol{H}_n^{(b)} \right)^{-1} \boldsymbol{H}_n^{(a)}$ has no nonpositive real eigenvalues.

Using the Edge Theorem, the controller is designed for the 4 vertices of the polytopic system, e.g. using the Neymark D-partition method that guarantees required closed-loop degree of stability. Then, stability of each edge of the box $\mathbf{Q}$ is checked by e.g. the Bialas Theorem. If any of the edges is unstable, new controller coefficients are to be designed.

2.2 Robust controller design using the Small Gain Theorem

Consider a perturbed plant with unstructured additive uncertainty in the form

$$G_p(s) = G_{nom}(s) + \partial G(s) \tag{8}$$

where $G_{nom}(s)$ is the nominal model and $\partial G(s)$ are additive uncertainties.

The nominal model can be obtained e.g. by N identifications of the plant (in N working points) by taking mean values of the nominator and denominator coefficients, respectively:

$$G_{nom}(s) = \frac{(B_1(s) + \ldots + B_N(s))/N}{(A_1(s) + \ldots + A_N(s))/N} \tag{9}$$

For each ω the uncertainties are found by substituting $s = j\omega - \alpha$, where α is the required stability degree:

$$\delta G(\omega) = \max \left| G_{nom}(s) - G_{p_i}(s) \right|_{s = j\omega - \alpha}, \text{ for } i = 1, \ldots, N \tag{10}$$

Theorem 3 (Small Gain Theorem)

Assume that the open-loop system is stable. The closed-loop system is stable if and only if the open-loop magnitude satisfies

$$\left| G_R(j\omega) G_p(j\omega) \right| < 1, \text{ for } \omega \in \langle 0, \infty \rangle \tag{11}$$

Theorem 4

Consider an auxiliary characteristic polynomial in the form

$$1 + F_{URO}(s) \frac{\delta G(s)}{G_{nom}(s)} \tag{12}$$

where

$$F_{URO}(s) = \frac{G_{nom}(s) G_R(s)}{1 + G_{nom}(s) G_R(s)}. \tag{13}$$

Assume that the open-loop system (nominal model and controller) and the auxiliary characteristic polynomial (12) are stable. Then closed-loop characteristic polynomial $p(s) = 1 + G_p(s) G_R(s)$ with unstructured additive uncertainties (8) is stable if and only if the following condition holds:

$$\left| F_{URO}(j\omega - \alpha) \right| < \frac{1}{\left| \dfrac{\delta G(\omega)}{G_{nom}(j\omega - \alpha)} \right|} = M_0(\omega) \text{ for } \omega \in \langle 0, \infty \rangle \tag{14}$$

Condition (14) is verified graphically. The robust controller design using Small Gain Theorem is realized in the following steps:

1. Specify the closed–loop system magnitude corresponding to the transfer function:

$$W(s) = \frac{G_{nom}(s) G_R(s)}{1 + G_{nom}(s) G_R(s)} \tag{15}$$

If the nominal model is of second order then $W(s)=\dfrac{as+1}{bs+1}$ and $G_R(s)=\dfrac{W(s)}{G_{nom}(s)-W(s)G_{nom}(s)}$ is a PID controller.

2. Choose the numerator of $W(s)$ equal to the numerator of G_{nom}

3. Choose $b > a$ and design the robust controller so that (14) is satisfied.

2.3 Robust controller design using the Genetic Algorithm

Consider $c = \{c_1, c_2, \dots , c_q\}$ to be the set of designed controller parameters and let $s = \{s_1, s_2, \dots, s_p\}$ is the set of parameters of the controlled system. During the operation of the plant, the parameters s_i can vary within some uncertainty domain

$$S : s_{i,\min} \le s_i \le s_{i,\max} \; ; i = 1,2,\dots, p \tag{16}$$

where $s_{i,min}$ and $s_{i,max}$ are the minimum and maximum possible values of the i-th system parameter, respectively. Consider W different (physical) working points of the controlled process, defined by different vectors s, which are to be controlled by the robust controller. For that case consider the cost function in the additive form

$$J = \sum_{i=1}^{W} J_i \tag{17}$$

comprising performance evaluation (for instance (18)) in all W working points. It is also recommended to include the measured noise from the real system or other possible disturbances or expected situations in the simulation model. Note, that alternatively to the set of W defined physical working points we can use a set of 2^p system parameter vectors located in the vertices of a polytope representing bounds of the parameter space S [5, 10].

Alternative to the previous method, the following method can be considered [11] for the working points selection. In each generation of the GA, n random working points (for all chromosomes of the population the same ones) are generated i.e. n vectors (say $n=100$) of system parameters s become random values from the domain S.

The controller design principle is actually an optimization task - search for such controller parameters from the defined parameter space, which minimize the performance index. The cost function (fitness) is a mapping $R^n \to R$, where n is the number of designed controller parameters. The cost function can to represent sum of absolute control errors (SAE) in following form:

$$J = \sum_{i=1}^{N} |e_i| \quad = \quad \sum_{i=1}^{N} |w_i - y_i| \tag{18}$$

where w is reference variable, y is controlled output, e is control error and N is number of patterns. Fitness is represented by the cost function or in the case of control, by the modified cost function, which can be penalized for example by derivation of process output y or by saturation of control action u. Modified cost function is in following form:

$$J = \sum_{i=1}^{N} |e_i| + \alpha \sum_{i=1}^{N} \left| \frac{dy_i}{dt} \right| + \beta \sum_{i=1}^{N} |u_c - u| \tag{19}$$

where u_c is control action from controller in front of saturation and α, β are weight constants.

The evaluation of the cost function consists of two steps. The first step is the computer simulation of the closed-loop time-response, and the second one is the performance index evaluation.

Genetic algorithms are described in e.g. [3-11] and others. Each chromosome represents a potential solution, which is a linear string of numbers, whose items (genes) represent in our case the designed controller parameters. Because the controller parameters are real-number variables and in case of complex problems the number of the searched parameters can be large, real-coded chromosomes have been used.

Without loss of generality let us consider a PID controller with feedforward structure, described in the continuous time domain by the equation (20), where P, I, D are controller parameters and t is a time.

$$u(t) = P.e(t) + I.\int_0^t e(t)dt + D.\frac{de(t)}{dt} \tag{20}$$

The searched PID controller parameters are $P \in R^+$, $I \in R^+$, $D \in R^+$. The chromosome representation in this case can be in form ***ch***$=\{P,I,D\}$.

A general scheme of a GA can be described by following steps (Figure 1):

1. Initialisation of the population of chromosomes (set of randomly generated chromosomes).

2. Evaluation of the cost function (fitness) for all chromosomes.

3. Selection of parent chromosomes.

4. Crossover and mutation of the parents $\rightarrow$ children.

5. Completion of the new population from the new children and selected members of the old population. Jump to the step 2.

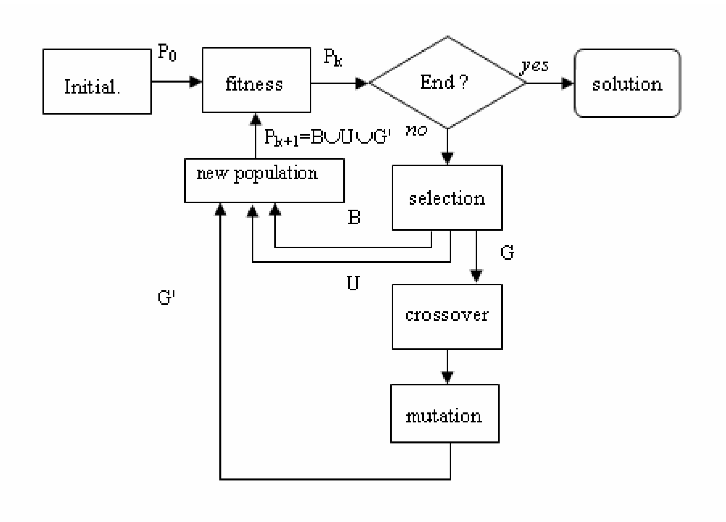

Figure 1: Block scheme of the used genetic algorithm

A block scheme of a GA-based design is in Figure 2. Before each cost function evaluation, the corresponding chromosome (genotype) is decoded into controller parameters of the simulation model (phenotype) and after the simulation the performance index is evaluated.

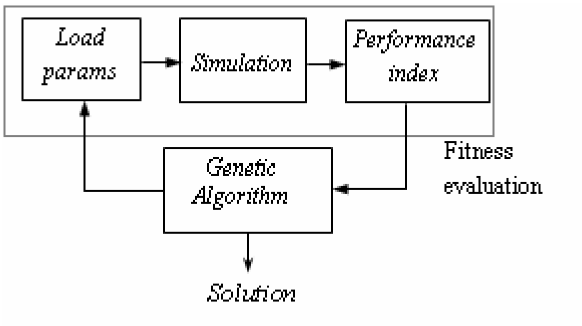

Figure 2: Block scheme of the GA-based controller design

3 Simulation results for robust PID controller design

The comparison of methods is illustrated by the robust controller design for two linear systems with uncertain parameters. The transfer functions of system A with uncertain parameters is in the following form:

$$G_s(s) = \frac{b_0}{a_2 s^2 + a_1 s + 1}$$

where b_0 is (0.5 – 4), a_2 is (0.05 – 1), a_1 is (0.45 – 2), control action u is saturation in range (-10, 10).

The transfer functions of system B with uncertain parameters is in the following form:

$$G_s(s) = \frac{b_1 s + b_0}{a_2 s^2 + a_1 s + 1}$$

where b_1 is (-0.1 – 0.3), b_0 is (1 – 5), a_2 is (1 – 4), a_1 is (0.1 – 2), control action u is saturation in range (-10, 10).

Step responses of systems A and B for boundary values of system parameters are depicted in Figure 3.

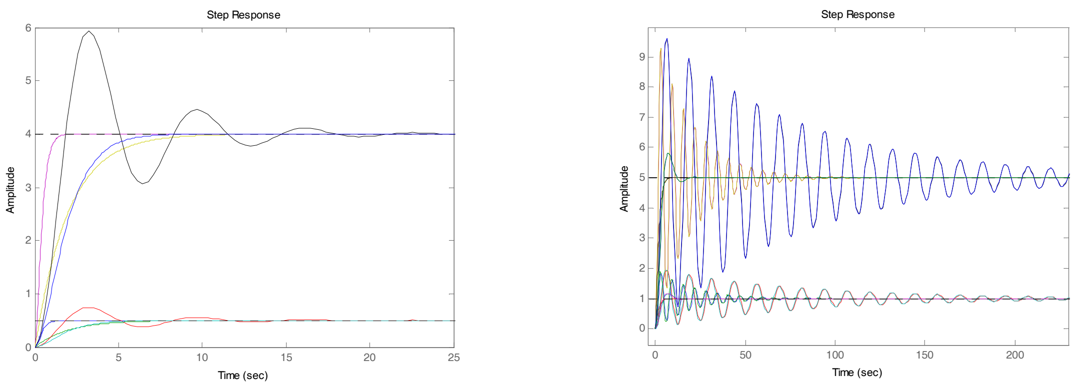

Figure 3: Step responses of systems A and B for boundary values of system parameters

Based on the design methods of robust controller described in the previous chapter, were designed of PID controller parameters for systems A and B. The proposed PID controller parameters are in Table 1.

TABLE 1: CONTROLLER PARAMETERS

Method	System A			System B		
	P	I	D	P	I	D
Edge	5	8	2	5.5	3	4
SGT	5.4	4.4	2.3	0.0705	0.0671	0.1678
GA	16.1502	3.7161	2.7860	14.6053	1.4754	4.7974

The proposed Statistical robustness measure (SRM) is based on statistical evaluation of a set of (more than 1000) closed-loop simulation experiments with randomly generated system parameters from the parameter box [11]. The SRM can be expressed by a scalar value calculated as

$$SRM = \frac{1}{N} \sum_{i=1}^{N} J_i \tag{21}$$

where N is the number of closed-loop simulation experiments and J is a selected performance index (18). In Table 2 the performance indices for selected methods in 1000 randomly generated system parameters from the parameter box are computed. Statistical robustness measure (SRM) according to equation (21), average values of overshoot and settling time are compared. Smaller values of the performance index represent a better closed-loop behavior. A more transparent evaluation of this experiment represents the use of the probability density function (Fig.6) (the probability vs. the performance index). The control performance is better for such controllers, for which the density function is located to the left within the horizontal axis range.

TABLE 2: CRITERION CONTROL QUALITY VALUES

Method	System A			System B		
	SRM	Overshoot [%]	Settling time [s]	SRM	Overshoot [%]	Settling time [s]
Edge	78.623	30.74	4.68	117.264	22.18	6.12
SGT	72.283	16.87	4.26	642.918	3.33	30.86
GA	32.655	3.57	3.04	55.155	5.49	4.31

The comparison of closed loop responses of system A (8 - boundary values of system parameters) for the genetic algorithm (GA) and Small Gain Theorem (SGT) design method are shown in Fig.4.

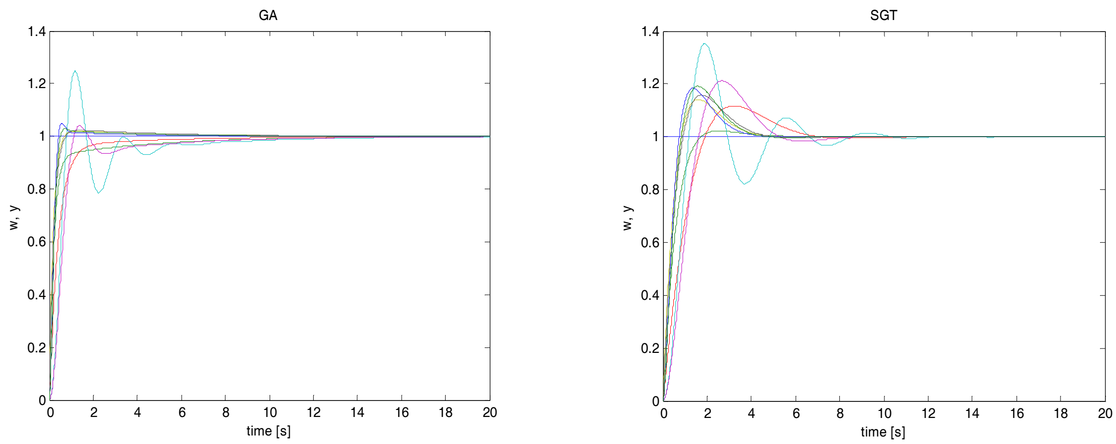

Figure 4: Comparison of closed loop responses of system A (8 system parameters) for the genetic algorithm and Small Gain Theorem design method

The comparison of closed loop responses of system B (16 - boundary values of system parameters) for the genetic algorithm (GA) and Edge Theorem design method are shown in Fig.5.

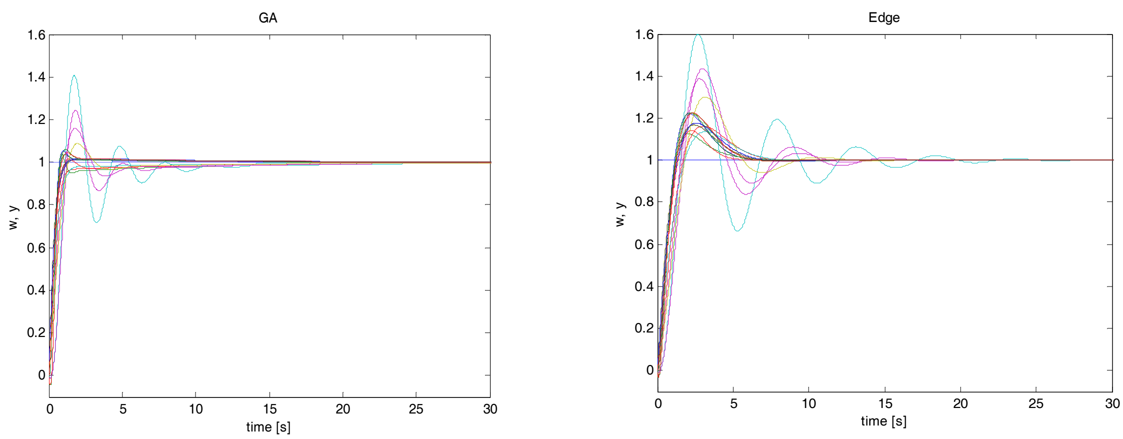

Figure 5: Comparison of closed loop responses of system B (16 system parameters) for the genetic algorithm and Edge Theorem design method

The probability of density functions for all methods are depicted in Fig.6.

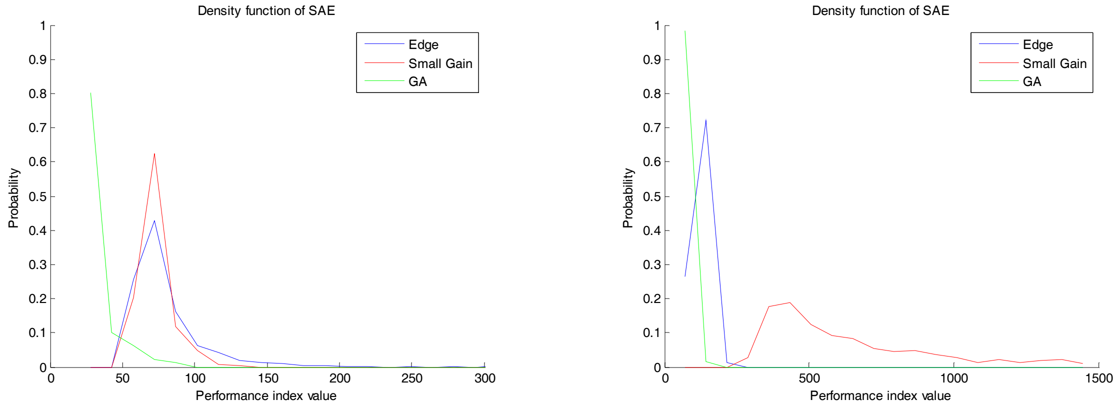

Figure 6: Probability of density function of performance index (SAE - sum of absolute control error)

4 Conclusion

The main aim of this paper has been to design robust controllers using Edge Theorem, Small Gain Theorem and genetic algorithm for uncertain SISO system. The comparison of these methods was illustrated by the robust controller design for two linear systems with uncertain parameters. For both systems better design of robust controller was realized using genetic algorithm. Design of robust controller using genetic algorithm provided better values of performance indexes in Table 2 and also density function of performance index had better shape. The design methods based on the Edge Theorem and the Small Gain Theorem guarantee the required closed-loop stability degree. Method based on genetic algorithm does not guarantee closed-loop stability, which should be verified by the statistical test in many (more than 1000) work points.

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