FREQUENCY CHARACTERISTICS VISUALIZATION IN Z-PLANE

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Abstract

The contribution is oriented but not limited to system characteristics of Finite impulse response (FIR) digital systems with linear phase characteristic. The properties of LTI system depend on values of the roots of transfer function. The visualization of mentioned characteristics helps to understand principles of behavior of discrete LTI systems.

1 System Characteristics of Digital Systems

Transfer function and impulse response are basic characteristics of digital linear time invariant (LTI) system. The properties of impulse response *h(n)* in time domain and frequency characteristics of the transfer function *H(z)* in z-plane depend on values of the roots of transfer function. To understand basic principles of behavior of discrete LTI systems the visualization of mentioned characteristics is profitable.

We are focused on design and the properties description of digital FIR and IIR filters, they are based on zeros and poles projection in z-plane. The user can display the frequency characteristics of discrete system transfer function $H(z)$ (1) - magnitude $|H(\Omega)|$ and phase $\varphi(\Omega)$ (2) frequency characteristics and impulse response $h(n)$ (3) as the basic characteristic in time domain

$$H(z) = a_0 \frac{\prod_{k=1}^{N}(z - z_{0k})}{\prod_{j=1}^{M}(z - z_{xj})} \tag{1}$$

where z_{0k} represent zeros and z_{xk} poles – roots of transfer function.

$$H(\Omega) = |H(\Omega)|.e^{j\varphi(\Omega)} \tag{2}$$

$$h(n) = Z^{-1}\{H(z)\} \tag{3}$$

Essentially, the Finite Impulse Response (FIR) filters with linear phase characteristics are addressed in our contribution. These systems have the special zeros distribution in z-plane, that offer symmetrical impulse response. In the case of selection of FIR filter design, each assigned zero-position of the transfer function is appended to desired roots combination (the complex conjugate and inverse roots) automatically. The correct zeros distribution can be observed by symmetrical *h(n)*. The other possibility of filter design are all-pass filters $H_{all}(z)$ (4).

$$H_{all}(z) = \frac{z^{-1} - \overline{z_0}}{1 - z_0 \cdot z^{-1}} \tag{4}$$

In this case the complex conjugate counterpart to each selected root is added automatically. It means, to each zero the pole is selected and vice versa.

2 Pole-Zero Editor Description

The filter is assigned in the form of the roots of transfer function *H(z)*, i.e. its zeros and poles in z-plane. The buttons described in Tab. 1 enable us to set or remove a new pole or zero complex values of *H(z)*. The values can be assigned in two ways – in Cartesian co-ordinates as real and imaginary part or in polar co-ordinates as magnitude and phase. The desired option is selectable in the main window in the right lower corner.

In Fig. 1 we can see the main window with four plots. On the left side, there are frequency characteristics – amplitude and phase, and on the right side, there are the impulse response and pole-zero diagram. New roots can be set also directly by clicking to the pole-zero diagram or changing their position using "Drag and Drop". After each new root selection or deletion the corresponding filter is designed and three other plots are generated automatically. In this way the corresponding properties of selected filter can by monitored.

Table 1: DESCRIPTION OF BUTTONS

0	addition of zero (the root in the nominator polynomial of transfer function) in z-plane
X	addition of pole (the root in the denominator polynomial of transfer function) in z-plane
00	addition of two (or four) zeros (the roots of FIR filter transfer function) in z-plane in the case of filter with linear phase characteristic
X0	addition of zero and pole in z-plane in the case of all-pass IIR filter

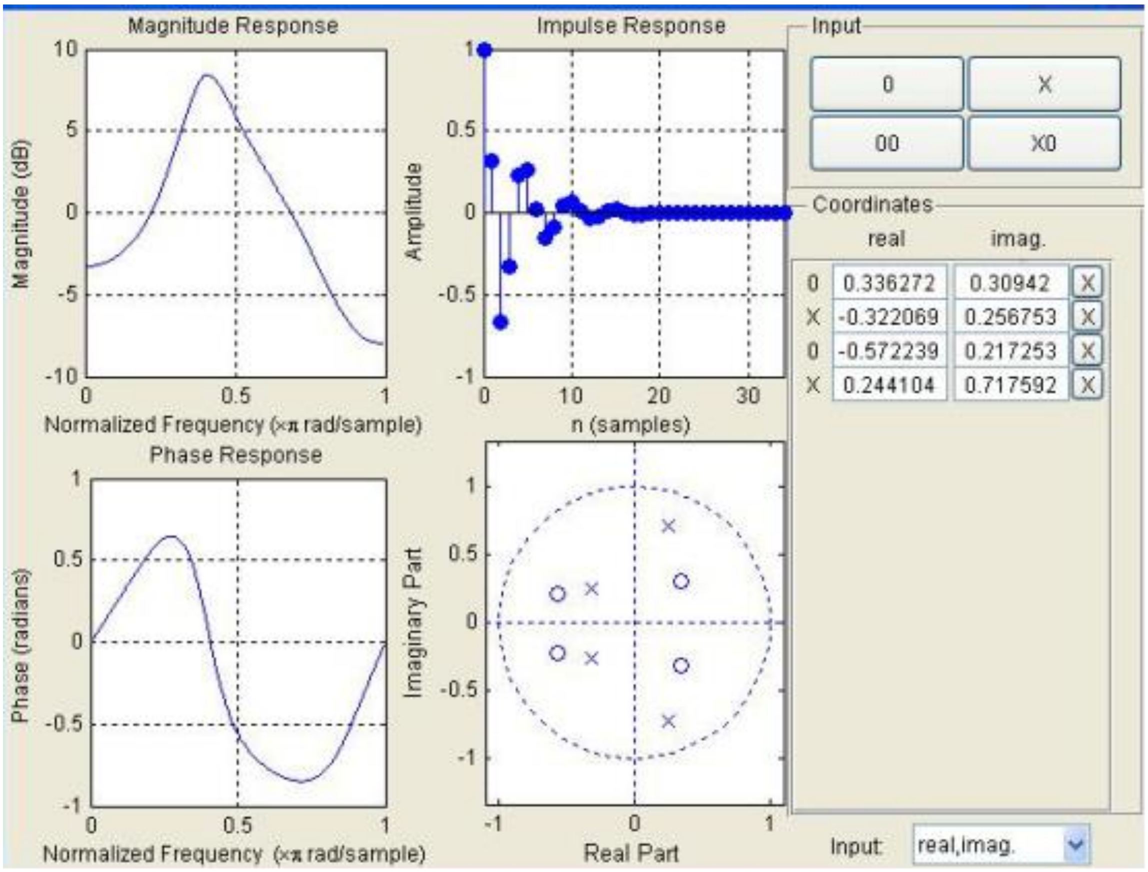

Figure 1: Visualization of impulse response and magnitude and phase characteristics for selected values of zeros and poles of transfer function in z-plane

3 Conclusion

We described basic characteristics of digital linear time invariant (LTI) system software product created in Matlab for visualization of digital filter characteristics. The product - the pole-zero editor (PZ-editor) enable us to design FIR and IIR filters and to keep the frequency characteristics and system stability under review interactively.

The visualization is helpful by the correct input (zeros and poles) interpretation, handling and evaluation of the output for designed transfer function $H(z)$.

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